2026

MATHEMATICS

(Minor)

Paper : MAT4200104MN

(Calculus)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

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1. Answer the following questions : 1×8=8

   (a) Define continuous functions.

   (b) State Rolle's theorem.

   (c) Find $f_x(0,1)$, where $f = x^2 + y^2$.

   (d) Evaluate $\lim_{x \to 0} \frac{1}{x}$.

   (e) Is every continuous function differentiable?

   (f) Find the value of $\int_a^a f(x)$.

   (g) State the Leibnitz's theorem.

   (h) Evaluate $\log \lim_{x \to 0} \frac{\tan x - x}{x^3}$.

2. Answer the following questions (any *six*) : 2×6=12

   (a) Show that $f(x) = |x|$ is continuous at $x = 0$.

   (b) If $f(x, y) = \sqrt{y+1} + \ln(x^2 - y)$, find $f(e, 0)$.

   (c) Evaluate $\int \tan^n x \, dx$.

   (d) Find y_n using successive differentiation of $y = (ax + b)^m$.

(e) In Leibnitz's theorem explain how we choose 1st and 2nd terms.

(f) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}; xy \neq 0$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$.

(g) If $\lim_{x\to\infty} \sqrt{(x+1)} = \infty$, $\lim_{x\to\infty} \sqrt{(x-1)} = \infty$, then show that $\lim_{x\to\infty} \left[\sqrt{(x+1)} - \sqrt{(x-1)}\right] = 0$.

(h) If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$.

(i) Expand $\log(x+a)$ in power of x by Taylor's theorem.

(j) Write the Maclaurin's expansion of $\sin x$.

3. Answer the following questions (any *four*) : 5×4=20

(a) Expand $\int \sin^n x\, dx$.

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(b) Find y_n of $y = \sin(ax+b)$.

(c) State and prove Euler's theorem of homogeneous functions.

(d) If $u = 3(lx+my+nz)^2 - (x^2+y^2+z^2)$ and $l^2+m^2+n^2 = 1$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

(e) Find $\lim_{n\to\infty}\left[\sum_{r=1}^{n} \frac{1}{2^r}\right]$, where [.] denotes the greatest integer.

(f) Evaluate $\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+4\right)\right\}^{\frac{1}{x}}$.

(g) State and prove Taylor's theorem.

(h) Expand $\log\sin x$ in power of $(x-2)$.

4. Answer the following questions (any *two*) : 10×2=20

(a) Show that $\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$, $0 < u < v$ and deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

(b) If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y}$, $x \neq y$ show that

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u$

(c) If $y = (x^2-1)^n$, prove that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

(d) Examine the continuity, the function f defined by

$$f(x) = \begin{cases} 2x+1, & 0 < x < \dfrac{1}{2} \\ 1, & x = \dfrac{1}{2} \\ 1-x, & \dfrac{1}{2} < x < 1 \\ 2x-2, & x \geq 1 \end{cases}$$

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At the points $x = \dfrac{1}{2}$ and $x = 1$.

(e) Expand $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x\, dx$, where m, n are positive integers greater than 1.

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