2026

MATHEMATICS

(Major)

Paper : MAT4200104MJ

(Calculus)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

(a) Define Homogeneous function.

(b) Find $f_{x(0,1)}$. Where $f = x^2 + y^2$.

(c) Define continuous function.

(d) State the Euler's theorem on Homogeneous functions.

(e) State true or false "Every differentiable function is continuous".

(f) Write Lagrange's Mean Value theorem.

(g) If $y = \frac{\log x}{x}$ find y_1.

(h) Find $\lim_{x \to 2} \frac{1}{2-x}$.

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2. Solve the following questions (any *six*) : 2×6=12

(a) If $\lim_{x \to 0} \left(2 + \frac{1}{x^2}\right) = \infty$, $\lim_{x \to 0} \left(1 + x + \frac{1}{x^2}\right) = \infty$ then show that $\lim_{x \to 0} (1 - x) = 1$.

(b) Evaluate $\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$

(c) Show that $f(x) = x|x|$ is derivable at the origin.

(d) Evaluate $\int (\log x)^n dx$

(e) Find the nth derivative of $y = e^{ax+b}$.

(f) Explain how we choose the 1st and 2nd terms in Leibnitz's theorem.

(g) Expand $\log(x+a)$ in power of x by Taylor's Theorem.

(h) Write the Maclaurin's expedition of $\sin x$ and $\cos x$.

(i) Verify Roll's theorem for $f(x) = x^2$ in $[-1, 1]$.

(j) If $u = \begin{bmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{bmatrix}$ find u_x.

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3. Solve the following questions (any *four*) : 5×4=20

(a) If $u = \cot^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{4}\sin 2u = 0$.

(b) Solve $\lim_{x\to 0}\left[\frac{\sin|x|}{|x|}\right]$, where [.] denotes greatest integer function.

(c) Find the n^{th} derivatives of $y = x^3 \cos x$, using Leibnitz's theorem.

(d) Expand $\sin x$ in power of $\left(x - \frac{\pi}{2}\right)$.

(e) If $u = \log\frac{x^2+y^2}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$.

(f) Evaluate $\lim_{x\to\infty}(\sqrt{x^2+x+1} - \sqrt{x^2+1})$.

(g) Using Lagrange's Mean Value theorem show that $\frac{1}{1+x} < \log(1+x) < x,\ x > 0$.

(h) Evaluate $\int \cos^n x dx$.

4. Solve the following questions (any *two*) : 10×2=20

(a) If $y = a\cos(\log x) + b\sin(\log x)$, then show that

(i) $x^2 y_2 + xy_1 + y = 0$

(ii) $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

(b) Show that the function $f(x) = |x| + |x-1| + |x-2|$ is continuous at the points $x = 0, 1, 2$.

(c) If $u = (x^3 + y^3 + z^3 - 3xyz)$ show that

(i) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{-3}{(x+y+z)^2}$

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(d) Evaluate $\int \sin^m x \cos^n x dx$, where m and n are positive integer greater than 1.

(e) State and prove Lagrange's Mean Value theorem.

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