Total number of printed pages–7

1 (Sem–4/AR) STA 05 (A)

Anandaram Dhekial Phookan College Library
Acc. No.
Date
Nagaon : Assam

2026

STATISTICS

(Major/Minor–1)

Paper : STA0400504

(Linear Algebra and System of Equations)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×8=8

(a) What is algebraic multiplicity of a characteristic root ?

(b) In a determinant, the number of rows is equal to the number of columns.
*(True **or** False)*

(c) If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$, find the cofactor of A.

(d) If $X = (1, 4, 7)$ and $Y = (4, 1, 7)$ then
(i) $X = Y$
(ii) $X \neq Y$
(iii) $X = 2Y$
(iv) $X = Y/3$
(Choose the correct option)

(e) 0 is the characteristic root of the matrix if and only if the matrix is _____ .
(Fill in the blank)

(f) If the non-singular matrix A is symmetric then A^{-1} is
(i) symmetric
(ii) skew symmetric
(iii) unit
(iv) zero *(Choose the correct option)*

(g) Obtain the matrix for the following quadratic form :

$$ax^2 + hxy + gxz + hyx + by^2 + fyz + gzy + fyz + cz^2$$

(h) Write the condition for the existence of a consistency of a system of equations.

2. Answer ***any six*** questions from the following : 2×6=12

(a) Find the value of the determinant of the matrix :

$$A = \begin{bmatrix} 4 & 7 & 8 \\ -9 & 0 & 0 \\ 2 & 3 & 4 \end{bmatrix}$$

(b) If A is a symmetric matrix, then prove that $adj\, A$ is also symmetric.

(c) State the condition under which a real quadratic form $\phi = X'AX$ will be
(i) positive definite
(ii) negative definite. 1+1=2

(d) Show that every super set of a linearly dependent set of vectors is linearly dependent.

(e) If A and B are two equivalent matrices, then show that rank A = rank B.

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(f) Show that the vectors $X_1 = (1, 2, 4)$ and $X_2 = (3, 6, 12)$ are linearly dependent.

(g) Write down the expansion of the determinant of order 4 along its first row.

(h) Show that if A is an orthogonal matrix then A^{-1} is also orthogonal.

(i) Define basis and dimension of a subspace.

(j) Write down the working rule for finding the solution of the equation $AX = B$.

3. Answer ***any four*** questions from the following : 5×4=20

(a) Prove that the rank of the transpose of a matrix is the same as that of the original matrix.

(b) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ to the normal form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ and hence determine the rank.

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(c) Prove that

$$\Delta \equiv \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)\left(a^2+b^2+c^2\right).$$

(d) Prove that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent.

(e) Determine the characteristic roots of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}.$$

(f) Solve completely the system of equations :

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0$$

(g) Find the inverse of the matrix

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Also verify the result.

(h) State *any five* properties of a determinants.

4. Answer ***any two*** questions from the following : 10×2=20

(a) Reduce the following quadratic form to canonical form and find its rank and signature :

$$x^2+4y^2+9z^2+t^2-12yz+6zx-4xy-2xt-6zt$$

(b) State Cayley Hamilton theorem. Use it to express $2A^5-3A^4+A^2-4I$ as a linear polynomial in A, when

$$A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}.$$

(c) Write the matrix form of the following system of equations :

$$\begin{aligned}2x-y+3z&=9\\ x+y+z&=6\\ x-y+z&=2\end{aligned}$$

and find A^{-1} if

$$A=\begin{bmatrix}2 & -1 & 3\\ 1 & 1 & 1\\ 1 & -1 & 1\end{bmatrix}$$

and hence solve the given equations.

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(d) Prove that elementary transformation do not change the rank of a matrix.

(e) Solve the following system of linear equation uisng Cramer's rule : 5+5=10

(i)
$$\begin{aligned}x+2y+3z&=6\\ 2x+4y+z&=7\\ 2x+2y+9z&=14\end{aligned}$$

(ii)
$$\begin{aligned}x+y+z&=7\\ x+2y+3z&=16\\ x+3y+4z&=22\end{aligned}$$