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1 (Sem–4/AR) STA 04 (A)

2026

STATISTICS

(Major/Minor–1)

Anandaram Dhekial Phookan College Library
Acc. No.
Date
Nagaon : Assam

Paper : STA0400404

(Mathematical Methods)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×8=8

(a) Define L-Hospital's Rule.

(b) The value of $\Delta^3 O^4$ is

(i) 1

(ii) 6

(iii) 0

(iv) None of the above

(c) What is the minimum value of $f(x)=|x-3|$?

(d) What do you mean by general solution of a differential equation ?

(e) What is another name of the Regula-Falsi method ?

(f) Identify the order of

$$y(n+2)+3y(n+1)-4y(n)=0.$$

(g) For a function to have a minimum value at a point, what must be the sign of the second derivative $\{f''(x)\}$?

(h) Stirling's formula is most accurate when n is

(i) Small

(ii) Large

(iii) Zero

(iv) Negative

2. Answer ***any six*** questions from the following : 2×6=12

(a) Solve the difference equation

$$U_{x+1}-U_x=Ca^x$$

(b) State Raabe's Test.

(c) Evaluate $\beta(1, n)$.

(d) Solve : $y(2xy+e^x)dx=e^x dy$

(e) Write down the Stirling's central difference formula.

(f) Show that $\frac{1}{1^P}-\frac{1}{2^P}+\frac{1}{3^P}-\frac{1}{4^P}+\ldots$ converges for $P>0$.

(g) Show that $f(x)=x-e^x$ has a maximum value and find it.

(h) Represent the following function in factorial notation :

$$f(x) = 2x^3 - 3x^2 + 3x - 10$$

(i) Write *any two* properties of Gamma function.

(j) If $u = x + y$ and $v = x - y$ find

$$\frac{\partial(u, v)}{\partial(x, y)}$$

3. Answer ***any four*** questions from the following : 5×4=20

(a) Define positive term series. Show that the positive term geometric series $1 + r + r^2 + \ldots$, converges for $r < 1$ and diverges to $+\infty$ for $r \geq 2$.

(b) Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$.

(c) Explain the Regula-Falsi Method of finding the root of a polynomial equation.

(d) Show that

$$\frac{\Delta^n O^m}{n!} = \frac{n\Delta^n O^{m-1}}{n!} + \frac{\Delta^{n-1} O^{n-1}}{(n-1)!}$$

(e) Deduce Bessel's Formula.

(f) Prove that

$$\beta(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$$

(g) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$ and $z = r\cos\theta$. Show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta .$$

(h) Find a real root of the equation $x^3 - 4x - 9 = 0$ using the Bisection method up to the fifth iteration.

4. Answer ***any two*** questions from the following : 10×2=20

(a) (i) Define Beta function and prove the symmetry property

$$\beta(m, n) = \beta(n, m)$$ 2+3=5

(ii) Evaluate

$$\int_0^\infty x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}}$$ 5

(b) (i) Write down the properties of maxima and minima and also their applications. 3+2=5

(ii) Show that minimum value of $u = xy + (a^3/x) + (a^3/y)$ is $3a^2$. 5

(c) (i) Show that, if n is a positive integer $\Gamma(n+1) = n!$. 5

(ii) Evaluate $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}\,dx$

using Gamma/Beta function. 5

(d) (i) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$. 5

(ii) State and prove the necessary condition for the convergence of an infinite series $\sum U_n$ and show that the converse is not true. 3+2=5

(e) (i) Find the general solution of the differential equation

$$x\frac{dy}{dx} + 2y = x^2 \log x.$$ 5

(ii) When a differential equation is said to be exact? Define integrating factor and homogeneous linear differential equation. 1+2+2=5

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