Total number of printed pages–8

1 (Sem–4/AR) STA 03 (A)

Anandaram Dhekial Phookan College Library
Acc.n No.
Date
Nagaon : Assam

2026

STATISTICS

(Major/Minor–1)

Paper : STA0400304

(Probability–2 and Probability Distribution–2)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×8=8

(a) For the geometric distribution variance is _____ than mean.

(Fill in the blank)

(b) If X and Y are two random variables

$$E(Y) = E\left[E(Y/X)\right]$$

*(State True **or** False)*

(c) The moment generating function of the sum of a number of independent random variables is equal to the product of their respective moment generating function.

*(State True **or** False)*

(d) Write the probability generating function of negative binomial distribution.

(e) What are the parameters of Cauchy distribution ?

(f) The ratio of two independent gamma variate is a beta variate of 2nd kind.

*(State True **or** False)*

(g) If $X_1, X_2, \ldots, X_n$ are independent and identically distributed (iid) continuous random variables with p.d.f. $f(x)$ and c.d.f. $F(x)$. What is the p.d.f. of the smallest order statistic $X_{(1)}$?

(Choose the correct option)

(i) $n[F(x)]^{n-1} f(x)$

(ii) $n[1-F(x)]^{n-1} f(x)$

(iii) $n[F(x)] f(x)$

(iv) $[1-F(x)]^n f(x)$

(h) Which one of these is a symmetrical distribution ?

(Choose the correct option)

(i) normal distribution

(ii) geometric distribution

(iii) gamma distribution

(iv) negative binomial distribution

2. Answer ***any six*** questions from the following : 2×6=12

(a) Obtain the moment generating function (m.g.f.) of geometric distribution.

(b) For a normal distribution the variance is 7. Find the mean deviation about the mean.

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(c) Under what condition negative binomial distribution may be regarded as the generalisation of geometric distribution and how?

(d) Show that the probability generating function of the sum of two independent random variables is equal to the product of their probability generating function.

(e) Derive the cumulative distribution function of $X_{(1)}$.

(f) If X is a random variable such that $E(X)=3$ and $E(X^2)=13$ use Chebychev's inequality to determine a lower bound for the probability $P[-2<x<8]$.

(g) Give the outline of Lognormal distribution and give its uses.

(h) Show that for the negative binomial distribution $p(x)=\left(-\frac{r}{x}\right)Q^{-r}\left(-\frac{P}{Q}\right)^{x}$;

$x=0, 1, 2, \ldots Q-P=1$

cumulant generating function is $-r\log\left[1-p\left(e^{t}-1\right)\right]$.

(i) If X and Y are two random variables prove that

$$Var(x)=Var\left[E(X/Y)\right].$$

(j) Define Cauchy distribution. Mention some applications of Cauchy distribution.

3. Answer ***any four*** questions from the following : 5×4=20

(a) Find mean and variance of the hypergeometric distribution.

(b) The joint probability density function of two-dimensional random variable (X, Y) is given by

$$f(x, y)=\begin{cases}2 & ; \ 0<x<1, 0<y<x \\ 0 & ; \ \text{otherwise}\end{cases}$$

(i) Find the marginal density functions of X and Y.

(ii) Find the conditional density function of Y given $X=x$ and conditional density function of X given $Y=y$.

(iii) Check for independence of X and Y.

2+2+1=5

(c) State and prove Chebyshev's Inequality.

(d) Prove that the mean deviation about mean in case of normal distribution is $\frac{4}{5}\Delta$.

(e) Show that the mean and variance of the geometric distribution

$p(x) = q^x p;\ x = 0, 1, 2, \ldots$

are respectively $\frac{q}{p}$ and $\frac{q}{p^2}$.

(f) Show that any linear combination of n independent normal variate is also a normal variate.

(g) State and prove the weak law of large numbers.

(h) Let $X_i (i = 1, 2, \ldots, n)$ is iid random variable having Weibull distribution. Show that the variable $Y = min(X_1, X_2, \ldots, X_n)$ also has Weibull distribution.

4. Answer the following questions : ***(any two)*** 10×2=20

(a) Write the p.m.f. of negative binomial distribution. Obtain moment generating function and cumulant generating function of the distribution and hence find its mean and variance. 1+3+4+1+1=10

(b) (i) Derive the joint probability distribution of $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}\ (r < s)$ based on a random sample of size n from the exponential distribution with parameter α. 5

(ii) A random sample of size 4 is drawn from the discrete uniform distribution

$P(X = x) = \frac{1}{6};\ x = 0, 1, 2, \ldots, 6$

Obtain the distribution function of the smallest and the largest order statistic. 5

(c) Define beta distribution of 1st kind. Find the mean and variance of beta distribution of 1st kind and beta distribution of 2nd kind. 2+4+4=10

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(d) (i) If joint probability density function

$$f(x, y) = 8xy, \quad 0 < x < 1$$
$$0 < y < 1$$

then find $E[Y/X = x]$ and $E[XY/X = x]$. 5

(ii) State and prove Lindeberg-Levy's central limit theorem. 5

(e) (i) If X and Y are independent gamma variate with parameter μ and ν respectively. Show that the variable $u = X + Y$, $z = \frac{X}{X+Y}$ are independent and that u is a $\gamma(\mu+\nu)$ variate and z is a $\beta_1(\mu, \nu)$ variate. 7

(ii) State and prove the additive property of Gamma distribution. 3

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