1 (Sem–4/FYUGP) PHY 44 MJ

2026

PHYSICS

(Major)

Anandaram Dhekial Phookan College Library * Nagaon : Assam *
Acc. No.
Date

Paper : PHY4400404MJ

(Mathematical Physics)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×5=5

(a) Write the two dimensional wave equation of a vibrating membrane.

(b) Find the period of the function $f(x)=2\sin x+4\cos x$.

(c) State Residue theorem.

(d) What do you mean by contraction of tensors ?

(e) Write the expression for mean and variance of Poisson distribution.

2. Answer ***any five*** of the following questions : 2×5=10

(a) Write the orthogonality condition of sine and cosine functions.

(b) Check whether the complex function $f(z)=|z|$ is analytic or not.

(c) Evaluate $\int_C (z-z^2)\,dz$, where C is upper half of the circle $|z|=1$.

(d) State the Dirichlet conditions for the existence of a Fourier series.

(e) Write the law of transformation for the tensor

(i) A^p_{qr}

(ii) B^{mn}_{ijk}

(f) Prove that the contraction of the tensor A^p_q is a scalar or invariant.

(g) What is singularity of an analytic function ? Find the singularity of the function $f(z)=\dfrac{z+3}{(z-1)(z-2)}$ and write the type of the singularity of this function.

(h) Expand the function $f(x)=x$ for $0<x<2\pi$ in Fourier series.

(i) Express Laplace's equation in two dimensional Cartesian form and write its solution for positive separation constant.

(j) A coin is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.

3. Answer ***any four*** of the following questions : 5×4=20

(a) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ under the following conditions :

$$y(0, t) = 0,\ y(l, t) = 0,\ y(x, 0) = a \sin\frac{\pi x}{l}$$

and $\frac{\partial y(x, 0)}{\partial t} = 0$.

(b) A function is defined as

$$f(x) = \begin{cases} 0 & \text{for } -\pi < x < 0 \\ h & \text{for } 0 \le x < \pi \end{cases}$$

is termed as a square-wave function. Obtain the Fourier series expansion.

(c) State and prove the Cauchy's integral formula for complex integration.

(d) Find the value of the integral $\int_0^{1+i} \left(x - y - ix^2\right) dz$ along real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.

(e) What is Kronecker delta ? Show that Kronecker delta is a mixed tensor of rank 2. 2+3=5

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(f) Using residue theorem evaluate the integral $\int_C \frac{4 - 3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$.

(g) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} + u$, where $u(x, 0) = 3e^{-4x}$.

(h) Express the Fourier series in complex form.

4. Answer ***any one*** of the following questions : 10×1=10

(a) Find the Fourier series expansion of

$$f(x)=\begin{cases}-\pi, & \text{for } -\pi < x < 0\\ x, & \text{for } 0 < x < \pi\end{cases}$$

Also show that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$

7+3=10

(b) *(i)* Expand the following function in Taylor series :

$$f(z)=\frac{1}{z+1}, \text{ about } z=1$$ 5

(ii) Show that $\int_0^{\infty}\frac{dx}{1+x^2}=\frac{\pi}{2}$

(using residue theorem) 5

(c) *(i)* What are symmetric and antisymmetric tensors ? Prove that any second rank tensor can be written as a sum of symmetric and antisymmetric tensor. 2+4=6

(ii) What is Levi-Civita tensor ? Prove that $\delta_{ij}\,\varepsilon_{ijk}=0$. 2+2=4

(d) *(i)* What is analytic function ? Deduce Cauchy-Riemann equation as a necessary condition for a complex function to be analytic. 1+6=7

(ii) Expand $f(z)=\frac{e^z}{(z-1)^2}$ in Laurent's series about $z=1$. 3

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