Total number of printed pages–7

1 (Sem–4/AR) PHY 02(A)

2026

Anandaram Dhekial Phookan College Library
Acc. No.
Date
Nagaon : Assam

PHYSICS

(Major/Minor–1)

Paper : PHY0400204

(Quantum Mechanics–I)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. Answer ***all*** the following questions : 1×5=5

(a) What do you mean by de Broglie hypothesis ?

(b) What are Dirac notations ?

(c) What are the eigenvalues of Hamiltonian operator ?

(d) Write down the expression for the energy quantum states of a linear harmonic oscillator.

(e) Explain the significance of canonical conjugate pairs.

2. Answer ***any five*** of the following questions : 2×5=10

(a) What are the characteristic features of the black body radiation spectrum ?

(b) Explain, what do you mean by Hilbert space.

(c) Verify the acceptability of the wave function

$$\psi(x) = x^2 .$$

(d) What is Hermitian operator ? Why is it so important in quantum mechanics ?

(e) Show that group velocity will be equal to that of the phase velocity provided that phase velocity is constant.

(f) Write down the time dependent Schrödinger equation for a free particle.

(g) Establish that time independent Schrödinger equation may be expressed as an eigenvalue equation.

(h) What do you mean by Gaussian wave packet ? Write the representation of its wave function.

Gyanendra Phookan College Library Acc. No. Date Nagaon : Assam

(i) Why do we prefer to express Schrödinger wave equation for the hydrogen atom in terms of spherical polar coordinates ?

(j) Show that for a pendulum of length $9{\cdot}8m$, the zero point energy is $\hbar/2$.

3. Answer ***any four*** of the following questions : 5×4=20

(a) Show that, if a proton and an electron have the same de Broglie wavelength, then the energy of the electron will be greater than that of the proton.

(b) Differentiate between photoelectric effect and compton effect. What is compton shift ? Calculate the numerical value of compton wavelength. 2+1+2=5

(c) Establish that both position operator and momentum operator are Hermitian operator. 2+3=5

(d) Show that

$$[\hat{E}, \hat{t}] = i\hbar .$$

What does this relation mean ? 4+1=5

(e) The wave function of a particle confined in an one-dimensional box of length L is

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}$$

in the range $0 < \pi < L$ and zero otherwise. Show that expectation value for the observable of momentum p in this state ψ is zero.

(f) What are stationary states ? Show that general solution of the time dependent Schrödinger equation may be expressed in terms of the linear combination of stationary states. 1+4=5

Bhekal Phookan College Library * Nagaon, Assam * Date

(g) Show that the ground state wave function

$$\psi(r) = \sqrt{\frac{1}{\pi a_0^3}}\, e^{-r/a_0}$$

of an electron in a hydrogen atom is a normalised one.

(h) What do you mean by degenerate states ? Show that for the hydrogen atom there are n^2 eigenfunctions for a given value of principal quantum number n. 1+4=5

4. Answer ***any one*** of the following questions : 10

(a) Consider about a wave packet formed by the superposition of two waves. Use this concept of wave packet in order to have an expression for Heisenberg's uncertainty principle.
Use this principle to show that electrons cannot exist inside the nucleus. 7+3=10

(b) (i) Explain the significance of the quantum numbers, n, l and m. Can the principal quantum number of electron in hydrogen atom be zero ? 3+1=4

(ii) Show that the most probable distance of the electron from the nucleus in the ground state of hydrogen atom is equal to the Bohr's radius a_0. 6

(c) (i) What is quantum dot ? The lowest energy of a particle enclosed in a box is $50eV$. Calculate the next three higher energies of the particles. 1+3=4

(ii) Write down the time independent Schrödinger equation for a simple harmonic oscillator of mass 'm' having potential energy

$$V(x)=\frac{1}{2}m\omega^2x^2$$

where 'ω' is the angular frequency. 2

(iii) Eigenfunction of Hamiltonian operator for the ground state is

$$\psi=\left(\frac{a}{\pi}\right)^{1/4}e^{-ax^2/2}$$

where $a=m\omega/\hbar$.

Use Schrödinger time independent equation is order to calculate the energy eigenvalue in the ground state of the oscillator. 4

Pandaram Dhekiajuli Pheokan College Library Acc. No. Date Assam

(d) (i) What do you mean by quantum tunnelling through barrier ? Discuss in brief, how can this process provide a solution to the observed phenomenon of alpha decay. 1+2=3

(ii) A particle is confined in an one dimensional potential well of the form

$$V(x)=0 \text{ for } 0\le x\le a$$

$V(x)$ is infinity for any other value of x. Obtain the energy levels and the corresponding normalised eigenfunctions of the particle. 4+3=7