Total number of printed pages–11

1 (Sem–4/AR) PHY 01 (A)

2026

PHYSICS

(Major/Minor–1)

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Acc. No.
Date
Nagaon : Assam

Paper : PHY0400104

(Classical Mechanics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

(a) If angular momentum of a particle is conserved which one of the following is true ?

(i) Force is zero

(ii) Linear momentum is zero

(iii) Impulse is zero

(iv) Torque is zero

(b) How many degrees of freedom and how many constraints does a simple pendulum have ?

(c) State the principle of virtual work.

(d) In the Lagrangian $L = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$, identify the cyclic coordinate and the conjugate momentum.

(e) If the Lagrangian is $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, which one is the correct Hamiltonian ?

(i) $H = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}k\dot{x}^2$

(ii) $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

(iii) $H = \frac{Px^2}{2m} - \frac{1}{2}kx^2$

(iv) $H = \frac{Px^2}{2m} + \frac{1}{2}kx^2$

(f) In the Lorentz transformation for reference frame, $x' = \gamma(x - vt)$, write down the expression for γ in terms of relative velocity.

(g) From the invariant space-time interval, $ds^2 = c^2dt^2 - \left(dx^2 + dy^2 + dz^2\right)$, write down the expression for space-like and time-like intervals :

(i) $ds^2 = 0, ds^2 < 0$

(ii) $ds^2 < 0, ds^2 > 0$

(iii) $ds^2 = 0, ds^2 > 0$

(iv) $ds^2 < 0, ds^2 = 0$

(h) For an inviscid flow, write down the Euler's equations of motion in vector form :

(i) $\rho\vec{g} - \vec{\nabla}p = \rho\dfrac{\partial\vec{v}}{\partial t}$

(ii) $\rho\vec{g} - \vec{\nabla}p = -\rho\dfrac{\partial\vec{v}}{\partial t}$

(iii) $\rho\vec{g} - \vec{\nabla}p = \left[\rho\dfrac{\partial\vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}\right]$

(iv) $\rho\vec{g} - \vec{\nabla}p = \rho\left[\dfrac{\partial\vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}\right]$

2. Answer ***any six*** questions : 2×6=12

(a) Express the constraint for a particle moving on a circular rim of radius r.

(b) At room temperature the vibrational degrees of freedom of an O_2 molecule are frozen out. Determine the number of degrees of freedom of the O_2 molecule at room temperature.

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(c) How action is defined in terms of Lagrangian? What is the value of the action according to the Hamilton's principle?

(d) If θ and z are cyclic coordinates in the Lagrangian $L = \dfrac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right)$, find the conjugate momenta corresponding to these cyclic coordinates.

(e) Lagrangian for a particle in an electromagnetic field is given by

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - q\phi(x, y, z, t)$$

$$+ q\vec{A}(x, y, z, t)\cdot\vec{v}.$$

If ϕ and $\vec{A}$ do not depend on x, then show that $m\dot{x} + qA_x$ is a conserved quantity.

(f) Construct the Hamiltonian for a one-dimensional simple harmonic oscillator.

(g) Write down the postulates of special theory of relativity.

(h) Show that the world line for a free particle is a straight line (use Lorentz transformation).

(i) Write down the expression for four velocity in component form.

(j) Write down the equation of continuity for an incompressible flow in differential form. What is the corresponding expression for a compressible fluid?

3. Answer ***any four*** questions : 5×4=20

(a) If R is the magnitude of the position vector for the centre of mass from an arbitrary origin, then show that

$$M^2R^2 = M\sum_i m_i r_i^2 - \frac{1}{2}\sum_{ij} m_i m_j r_{ij}^2 .$$

(b) Derive Euler-Lagrange equation from Hamilton's action principle.

(c) The Lagrangian for the planetary motion of a celestial body of mass m is given by

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$$

Construct the Lagrange equations of motion. Show that the canonical momentum is a constant. 3+2=5

(d) Two blocks of mass m and M connected by a massless spring of spring constant k are placed on a smooth horizontal table. Determine the equations of motion using Lagrangian mechanics.

(e) The Lagrangian for the planetary motion of a system of reduced mass μ is given by

$$L = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$$

Assuming $u = \frac{1}{r}$, derive the equation of the orbit given by

$$\frac{d^2u}{d\theta^2} + u = -\frac{\mu}{l^2u^2}F\left(\frac{1}{u}\right),$$

where l is the angular momentum.

(f) A particle of mass m moves on the x-axis under the force field whose potential energy is

$V = x(x^2 - 9x + 24)/3$. Show that there is a single position of stable equilibrium. Find the approximate period of small oscillations about this point. 2½+2½=5

(g) Explain why a moving clock ticks more slowly than a clock at rest. At what speed does a clock move if it runs at a rate which is one-half the rate of a clock at rest? Use time dilation to arrive at the results. 2+3=5

(h) The velocity distribution for a steady, incompressible laminar flow through a straight circular tube of radius R is given by

$$V_z = \frac{1}{4\mu}\left(\frac{\partial p}{\partial z}\right)\left(r^2 - R^2\right),$$

where the symbols have their usual meaning.

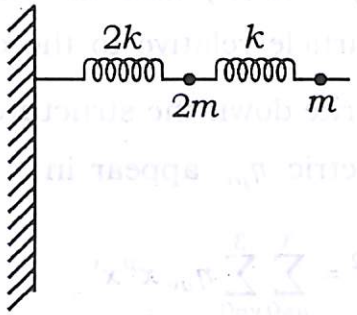
Derive the Poiseuills's law from the volume rate of flow. Find the expression for maximum velocity. 3+2=5

4. Answer ***any two*** questions : 10×2=20

(a) Write down the radial equation of motion for a central force. Using $\dot{\theta} = l/\mu r^2$, find the expression for total energy in terms of effective potential. Using inverse-square law of force, show that the trajectory is a conic section. Show that the eccentricity of the conic section is

$$e = \sqrt{\left(1 + \frac{2El^2}{\mu k^2}\right)} \qquad 2+3+3+2=10$$

(b) Two beads of mass $2m$ and m are connected to a fixed wall with two springs of spring constants $2k$ and k. If the masses can move without friction along a horizontal wire, find the eigenfrequencies of small amplitude oscillations.



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(c) Derive the expression for time dilation using Lorentz transformation. An atomic clock placed in a jet airplane measures a time interval of $3600 sec$ when the jet moves at a speed of $2000 m/s$. How much longer a time interval does an identical clock held by an observer at rest on the ground measure?

At what speed does a meter stick move if its length is observed to shrink to $0{\cdot}5m$? Use length contraction formula to arrive at the result. 3+4+3=10

(d) Derive the expression for relativistic velocity components. If two cosmic-ray protons approach Earth from opposite directions at speeds $v_1 = 0{\cdot}6c$ and $v_2 = -0{\cdot}8c$, find the velocity of each particle relative to the other.

Write down the structure of Minkowski metric $\eta_{\mu\nu}$ appear in

$$s^2 = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} \eta_{\mu\nu}\, x^\mu x^\nu .$$ 4+3+3=10

(e) For an incompressible Newtonian fluid, write down the expressions for normal and shearing stress. Hence derive the Navier-Stokes equation. 5+5=10

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