*Total number of printed pages–7*

**1 (Sem–4/FYUGP) MAT 42 MJ**

Anandaram Dhekial Phookan College Library
Acc. No.
Date
Nagaon : Assam

**2026**

# MATHEMATICS

( Major )

Paper : MAT4400204MJ

**( Complex Analysis–I )**

*Full Marks : 45*

Time : 2 hours

*The figures in the margin indicate full marks for the questions.*

1. Answer the following questions : 1×5=5

(a) Find $\lim_{z \to i} \frac{iz^3 - 1}{z + 1}$.

(b) A composition of continuous function is

(i) pointwise continuous

(ii) discontinuous

(iii) itself continuous

(iv) None of the above

*(Choose the correct answer)*

(c) The value of $Log(-ei)$ is

(i) $\pi/2 - i$

(ii) $i$

(iii) $1 + i$

(iv) $1 - \frac{\pi}{2} i$

*(Choose the correct answer)*

(d) The power expression of $\cos z$ is

(i) $\frac{e^{iz} + e^{-iz}}{2}$



(ii) $\frac{e^{z} + e^{-z}}{2}$

(iii) $\frac{e^{iz} + e^{-iz}}{2i}$

(iv) None of the above

*(Choose the correct answer)*

(e) What is Jordan arc ?

2. Answer ***any five*** of the following questions : 2×5=10

(a) If $f(z) = \frac{z^2}{z}$ $(z \neq 0)$, where $z = x + iy$, then write $f(z)$ in the form $f(z) = u(x, y) + iv(x, y)$.

(b) Show that : $\lim_{z \to \infty} \frac{2z^3 - 1}{z^2 + 1} = \infty$

(c) If $f(z) = z - \bar{z}$; then show that $f'(z)$ does not exist at any point.

(d) Find the singular point of

$$f(z) = \frac{z^2 + 1}{(z + 2)(z^2 + 2z + 2)}.$$

(e) If $e^z = 1+\sqrt{3}i$ then find $z$ in form of $z = x+iy$.

(f) Find the principal value of $(-i)^i$.

(g) Evaluate : $\int_1^2 \left(\frac{1}{t}-i\right)^2 dt$

(h) Evaluate : $\int_C \frac{exp(2z)}{z^4} dz$, where 'C' is the positively oriented unit circle $|z|=1$.

(i) If $f(z)=\frac{z}{\bar{z}}$, find $\lim_{z\to 0} f(z)$, if it exists.

(j) Write the function $f(z)=z+\frac{1}{z}(z\neq 0)$ in the form $f(z)=u(r,\theta)+iv(r,\theta)$.

3. Answer ***any four*** of the following questions : 5×4=20

(i) Sketch the set $|z-2+i|\leq 1$ and determine its domain.



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(ii) Show that the three cubic roots of $-8i$ lie at the vertices of an equilateral triangle that is inscribed in a circle of radius 2 centred at the origin.

(iii) If $z_1$ and $z_2$ are complex numbers then show that $\sin(z_1+z_2)=\sin z_1\cos z_2+\cos z_1\sin z_2$.

(iv) Suppose that a function $f(z)$ is analytic at a point $z_0=z(t_0)$ on a differentiable arc $z=z(t)(a\leq t\leq b)$. Show that if $w(t)=f(z(t))$ then $w'(t)=f'(z(t))z'(t)$ when $t=t_0$.

(v) Let 'C' be the arc of the circle $|z|=2$ from $z=2$ to $z=2i$, that lies in the 1st quadrant, then show that

$$\left|\int_C \frac{z-2}{z^2+1} dz\right| \leq \frac{4\pi}{15}.$$

(vi) Using anti-derivative, evaluate the integral $\int_C z^{1/2} dz$, where 'C' is a contour from $z=-3$, to $z=3$ that except for its end points, lies above the $x$-axis.

(vii) Show that $exp(2 \pm 3\pi i) = -e^2$.

(viii) If $f(z) = \frac{z+2}{z}$ and $C$ is the semi circle $z = 2e^{i\theta}, (0 \le \theta \le \pi)$, then evaluate $\int_C f(z)\,dz$.

4. Answer ***any one*** of the following questions : 10

(a) State and prove Cauchy-Riemann equations of an analytic function in polar form.

(b) (i) Let $u$ and $v$ denote the real and imaginary components of the function '$f$' defined by the equations :

$$f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, & \text{when } z \neq 0 \\ 0, & \text{when } z = 0 \end{cases}$$

Verify the Cauchy-Riemann equations at the origin $z = (0, 0)$. 5



(ii) Show that if a function $f(z) = u(x, y) + iv(x, y)$ and its conjugate $\overline{f(z)}$ are both analytic in a domain $D$, then $f(z)$ must be constant throughout $D$. 5

(c) State and prove the fundamental theorem of algebra.

(d) If a function '$f$' is analytic at all points interior to and on a simple contour '$C$', then prove that $\int_C f(z)\,dz = 0$.

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