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1 (Sem–4/FYUGP) MAT 44 MJ

2026

MATHEMATICS

(Major)

Anandaram Dhekial Phookan College Library * Nagaon : Assam *
Acc. No.
Date

Paper : MAT4400404MJ

(Number Theory–I)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

(a) State the Archimedean property for integers.

(b) Which of the following equations cannot be solved ?

(i) $6x+51y=22$

(ii) $33x+14y=115$

(iii) $14x+35y=93$

(c) State the Goldbach conjecture.

(d) Which one of the following is true for the congruence $6x \equiv 2 (mod 4)$?

(i) There are three distinct solutions

(ii) There is no solution

(iii) There are two distinct solutions

(e) Which of the following sets are complete residue system modulo m ?

(i) $1, 2, \ldots, m-1$

(ii) $2, 3, \ldots, m+1$

(iii) $0, 1, 2, \ldots, m$

(f) Which theorem states that "If p is a prime, then $(p-1)! \equiv -1 (mod\ p)$" ?

(i) Dirichlet's theorem

(ii) Wilson's theorem

(iii) Euler's theorem

(g) Euler's phi function of 1000 is –

(i) 100

(ii) 250

(iii) 400

(h) Mobius μ-function of 28 is –

(i) 1

(ii) 0

(iii) –1

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2. Answer ***any six*** of the following questions : 2×6=12

(a) Prove that there is no natural number between 0 and 1.

(b) If a prime p divides ab, then prove that p divides *either* a *or* b.

(c) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then prove that $ac \equiv bd \pmod{m}$.

(d) Solve $6x \equiv 3 \pmod{21}$.

(e) Find the highest power of 5 that divide 518 !.

(f) If p is a prime and $k > 0$, then prove that $\phi(p^k) = p^k\left(1 - \frac{1}{p}\right)$.

(g) Find $\sigma(40)$ and $\tau(40)$.

(h) Define the greatest integer function and find the value of $\left[-\frac{3}{2}\right]$.

(i) Express (726, 275) in the form of $m \times 726 + n \times 275$.

(j) If $a \mid c$, $b \mid c$ and $(a, b) = 1$, then prove that $ab \mid c$. Also prove that the result is false if $(a, b) \neq 1$.

3. Answer ***any five*** of the following questions : 5×4=20

(a) Determine all the positive solutions of $5x + 3y = 52$.

(b) Prove that there are infinitely many primes of the form $4q + 3$.

(c) Prove that 89 divides $2^{44} - 1$.

(d) Find the remainder when $1! + 2! + 3! + \ldots\ldots + 100!$ is divided by 12.

(e) State and prove the Euler's theorem.

(f) Prove that for the integer $n > 1$, then sum of the positive integers less than n and relatively prime to n is $\frac{1}{2} n\phi(n)$.

(g) If $\mu(n)$ is a multiplicative function then prove that

$$\Sigma\mu(d) = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } n > 1 \end{cases}$$

(h) If $n = p_1^{k_1} p_2^{k_2} \ldots\ldots p_r^{k_r}$ is the prime factorization, then prove that

$$\tau(n) = (k_1 + 1)(k_2 + 1)\ldots(k_r + 1).$$

4. Answer ***any two*** of the following questions : 10×2=20

(a) (i) By Mathematical induction establish the formula

$$1^2 + 2^2 + 3^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

for all $n \geq 1$. 5

(ii) Using Euclidean algorithm find the greatest common divisor of 341 and 527. 5

(b) (i) State and prove the fundamental theorem of Arithmetic. 6

(ii) If $2^k - 1$ is prime, then prove that k itself must be a prime. 4

(c) State and prove the Chinese Remainder theorem.

Or

(d) (i) State and prove the Fermat's little theorem. 1+4=5

(ii) Prove that if p is a prime, then $(p-1)! \equiv -1 \pmod{p}$. 5

(e) Solve the following set of simultaneous congruences :

$$2x \equiv 3 \pmod 5$$
$$4x \equiv 2 \pmod 6$$
$$3x \equiv 2 \pmod 7$$ 10

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