2026

MATHEMATICS

(Major)

Paper : MAT0600404

(Mechanics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

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1. Answer the following questions : 1×8=8

(a) Two equal forces act on a particle. Find the angle between the forces when the square of their resultant is equal to three times of their product.

(b) What is the physical significance of moment of a force about a point?

(c) Give one example of a couple from daily life.

(d) Define amplitude of a simple harmonic motion.

(e) What is the centre of gravity of a uniform triangular lamina?

(f) Define limiting equilibrium and limiting friction.

(g) State Newton's law of motion from which we derive the method of measuring force.

(h) Define impulsive force with an example.

2. Answer the following questions (any *six*) : 2×6=12

(a) P and Q are like-parallel forces. If P is moved parallel to itself through a distance x, then show that the resultant of P and Q moves through a distance $\frac{Px}{P+Q}$.

(b) Prove that the algebraic sum of the moments of the forces forming a couple about any point in their plane is a non-zero constant and equal to the moment of the couple.

(c) If the radial and transverse velocities of a particle are always proportional to each other, then show that the path is an equiangular spiral.

(d) Find the position of centre of gravity of a uniform semicircular arc of radius a.

(e) A point in a straight line with SHM has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is

$$2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

(f) State the laws of static friction.

(g) Write down the general conditions of equilibrium of a system of coplanar forces acting on a rigid body.

(h) A particle falls under gravity, supposed constant, in a resisting medium whose resistance varies as the square of the velocity. Find the terminal velocity of the motion.

(i) Prove that the change of kinetic energy of a body is equal to the work done by the acting force.

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(j) Find the magnitude of the force which, acting on a mass of 18 kg for 10 seconds, will generate in it a velocity of 5 km per hour.

3. Answer the following questions (any *four*) : 5×4=20

(a) Forces of magnitudes P, $2P$, $-P$, $2P$ act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ of the square $ABCD$, and a force of magnitude $P\sqrt{2}$ acts along each of $\overrightarrow{BD}$ and $\overrightarrow{CA}$. Show that the forces reduce to a couple of moment $2aP$, where a is the side of the square.

(b) Establish the following statement :

Any system of coplanar forces acting on a rigid body can ultimately be reduced either to a single force or to a single couple unless it is in equilibrium.

(c) A particle moves along a straight line, and at a distance x from a fixed point O on the line its velocity is $\mu\sqrt{\frac{c-x}{x}}$. Prove that its acceleration is directed towards O and is inversely proportional to the square of the velocity.

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(d) Find the CG of the area of the cardioid $r = a(1+\cos\theta)$.

(e) A particle of mass m falls from rest at a height h above the ground. Show that the sum of its potential and kinetic energies is constant throughout the motion.

(f) A shot of mass m is fired from a gun of mass M with velocity u relative to the gun. Show that the actual velocities of the shot and the gun are $\frac{Mu}{M+m}$ and $\frac{mu}{M+m}$ respectively.

(g) A uniform ladder rests in limiting equilibrium with lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If θ be the inclination of the ladder to the vertical, then prove that $\tan\theta = 2\mu$, where μ is the coefficient of friction.

(h) A particle of mass m is projected vertically upwards under gravity, the resistance of air being mk times the velocity. Show that the greatest height attained by the particle is

$$\frac{V^2}{g}[\lambda - \log(1+\lambda)]$$

where V is the terminal velocity of the particle and λV is the initial velocity.

4. Answer the following questions (any *two*) : 10×2=20

(a) State and prove Lami's theorem.

Forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ acting along $\vec{IA}$, $\vec{IB}$, $\vec{IC}$, where I is the incentre of the triangle ABC, are in equilibrium. Show that

$$P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$$

1+4+5=10

(b) Prove that in polar coordinates (r, θ), the velocity $\vec{V}$ and acceleration $\vec{a}$ of a particle moving in a plane curve are given by

$$\vec{V} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

10

(c) *(i)* Forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x\cos\theta + y\sin\theta = p$. Find the magnitude of the resultant and equation of its line of action. 5

(ii) A heavy uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and being

placed upon a peg at a distance b from the wall. Show that the inclination of the rod to the horizon θ is given by $\cos^3\theta = \frac{b}{a}$. 5

(d) *(i)* Find the centre of gravity of the solid formed by revolution of the quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about its minor axis. 5

(ii) A particle falls under gravity in a resisting medium whose resistance varies as the square of the velocity. Find the velocity of the particle at time t. 5

(e) *(i)* A particle is describing a plane curve. If the tangential and normal accelerations are each constant throughout the motion, then prove that the angle ψ, through which the direction of motion turns in time t, is given by $\psi = A\log(1 + Bt)$. 4

(ii) A particle of mass m is acted upon by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the

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origin O. If it starts from rest at a distance a from O, then show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 6

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