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3 (Sem–6/CBCS) MAT HC 2

2026

MATHEMATICS

(Honours Core)

Paper : MAT–HC–6026

(Partial Differential Equations)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** : 1×7=7

   (i) The partial differential equation is elliptical if

   (a) $B^2 - 4AC > 0$

   (b) $B^2 - 4AC \geq 0$

   (c) $B^2 - 4AC \leq 0$

   (d) $B^2 - 4AC < 0$

   (Choose the correct answer)

(ii) Fill in the blanks :

A solution $z = z(x, y)$ when interpreted as a surface in 3-dimensional space is called _____ _____.

(iii) State *True* **Or** *False* :

$xu_x + yu_y = u^2 + x^2$ is a semi-linear partial differential equation.

(iv) The characteristic equation of $yu_x + xu_y = u$ is

(a) $\frac{dx}{x} = \frac{dy}{y} = \frac{du}{u}$

(b) $\frac{dx}{y} = \frac{dy}{x} = \frac{du}{u}$

(c) $\frac{dx}{u} = \frac{dy}{x} = \frac{du}{y}$

(d) None of the above

(Choose the correct answer)

(v) Fill in the blanks :

$$\left(\frac{\partial z}{\partial x}\right)^2 + 2\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + z = 0$$

is a _____ _____ order partial differential equation.

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(vi) State *True* **Or** *False* :

The equation

$$u_{xx} + u_{yy} + u_{zz} = 0$$

is an Hyperbolic equation.

(vii) Which method can be used for finding the complete solution of a non-linear partial differential equation of first order

(a) Jacobi method

(b) Charpit's method

(c) Both (a) and (b)

(d) None of the above

(Choose the correct answer)

(viii) Find the degree and order of the equation

$$\frac{\partial^3 z}{\partial x^3} + \left(\frac{\partial^3 z}{\partial x \partial y^2}\right)^2 + \frac{\partial z}{\partial y} = \sin(x + 2y).$$

(ix) How many minimum no. of independent variables does a partial differential equation require ?

(x) The equation of the form $P_p + Q_q = \mathbb{R}$ is known as

(a) Charpit's equation

(b) Lagrange's equation

(c) Bernoulli's equation

(d) Clairaut's equation

(Choose the correct answer)

2. Answer ***any four*** : 2×4=8

(i) What is the classification of the equation ?

$$u_{xx} - 4u_{xy} + 4u_{yy} = e^y$$

(ii) Mention when Charpit's method is used. Name a disadvantage of Charpit's method.

(iii) Explain the essential features of the "Method of separation of variables".

(iv) Solve the differential equation $p + q = 1$.

(v) Determine whether the given equation is hyperbolic, parabolic or elliptic

$$u_{xx} - 2u_{yy} = 0.$$

(vi) Eliminate the constants a and b from $z = (x+a)(y+b)$.

(vii) Show that a family of spheres $(x-a)^2 + (y-b)^2 = r^2$ satisfies the partial differential equation $z^2\left(p^2 + q^2 + 1\right) = r^2$.

(viii) Define quasi-linear partial differential equation and give *one* example.

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3. Solve ***any three*** : 5×3=15

(i) Determine the general solution of $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$.

(ii) Apply the method of separation of variables $u(x, y) = f(x)\,g(y)$ to solve the equation $u_x + u = u_y$, $u(x, 0) = 4e^{-3x}$.

(iii) Reduce the equation $yu_x + u_y = x$ to canonical form and obtain the general solution.

(iv) Find a complete integral of the equation $x^2p^2 + y^2q^2 = 1$.

(v) Find the solution of the equation $z = pq$ which passes through the parabola $x = 0$, $y^2 = z$.

(vi) Find the integral surface of the linear partial differential equation

$$x\left(y^2 + z\right)p - y\left(x^2 + z\right)q = \left(x^2 - y^2\right)z$$

which contains the straight line $x + y = 0$, $z = 1$.

(vii) Solve :

$$y^2 p - xyq = x(z - 2y)$$

(viii) Form a partial differential equation by eliminating arbitrary functions f and F from $y = f(x - at) + F(x + at)$.

4. Answer ***any three*** : 10×3=30

(i) Classify the second order linear partial differential equation with example.

(ii) Find the canonical form of the equation $y^2 u_{xx} - x^2 u_{yy} = 0$.

(iii) Find the solution of the equation

$$z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$$

which passes through the x axis.

(iv) Use $v = ln\ u$ and $v = f(x) + g(y)$ to solve the equation

$$x^2 u_x^2 + y^2 u_y^2 = u^2.$$

(v) Solve :

$$(mz - ny)p + (nx - lz)q = ly - mx$$

(vi) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z.$$

(vii) Solve $z^2 = pqxy$ by Charpit's method.

(viii) Solve $(p^2 + q^2)y - qz = 0$ by Jacobi method.

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