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1A (Sem-1/ITEP) PHY01 MJ

2025

PHYSICS

(Major)

Paper : PHY0100104-N

(Mathematical Physics and Mechanics)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 5 = 5$
 - (a) What is the value of the curl of the gradient of a scalar field ϕ .
 - (b) If $\delta(x)$ is an even function of x , then find the value of $\delta(-x)$.
 - (c) Define gravitational potential.
 - (d) What is the difference between elastic and inelastic collision?
 - (e) State work-energy theorem.

2. Answer the following questions : **(any five)**
2×5=10

(a) Find $\nabla\phi$ if $\phi = \frac{1}{r}$, \vec{r} being the position vector.

(b) Show that the vector field
 $\vec{F} = 2x(y^2 + z^3)\vec{i} + 2x^2y\vec{j} + 3x^2z^2\vec{k}$
is conservative.

(c) What is the value of
 $\int_{-\infty}^{\infty} f(x)\delta(x-a)dx$? Evaluate
 $\int_{-\infty}^{\infty} e^{-5t}\delta(t-2)dt$.

(d) Three particles of masses 1kg, 2kg, and 3kg are placed at 0m, 2m, and 4m respectively. Find the centre of mass.

(e) Define Impulse.

(f) State the theorem of perpendicular axes.

(g) Why is friction considered as non-conservative force ?

(h) State the limitations of Hooke's law.

(i) A circular disc of mass 0.5kg and radius 0.1m is making 60 revolutions per minute about an axis passing through its centre and perpendicular to its plane. Calculate its kinetic energy.

(j) On what factors does the twisting torque of a wire depend ?

3. Answer the following questions : **(any four)**
5×4=20

(a) Define gradient of a scalar function. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction of $(2\vec{i} - \vec{j} - 2\vec{k})$.

(b) Define solenoidal and irrotational vectors. Prove that
 $(y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$
is both solenoidal and irrotational.

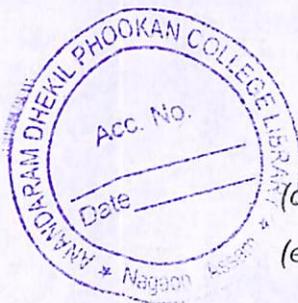
(c) (i) If
 $\vec{A} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$,
evaluate the line integral $\oint \vec{A} \cdot d\vec{r}$
from (0, 0, 0) to (1, 1, 1) along the curve C and $x = t$, $y = t^2$, $z = t^3$.

(ii) Prove that $\text{div curl } \vec{A} = 0$.
3+2=5

(d) Prove that $\nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B}$.

(e) A body moving in central field follows a trajectory given by $r = a \cos \theta$. Find the law of force.

(f) Derive the expression for fictitious forces which comes into play in uniformly rotating frame of reference.



(g) In case of an inelastic collision between two bodies in one dimension, obtain the expression for final velocity and the loss of kinetic energy.

(h) Derive Poiseuille's equation for the steady flow of a liquid through a capillary tube.

4. Answer the following questions : **(any one)**
10×1=10

(a) Define Stokes' theorem and mention its significance. Evaluate

$$\int_C [(2x - y) dx - yz^2 dy - y^2 z dz]$$

where C is the circle $x^2 + y^2 = 1$, corresponding to the surface sphere of unit radius. 3+2+5=10

(b) Prove that a cylindrical coordinate system is orthogonal. Represent the vector $\vec{A} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates. 5+5=10

(c) Derive the expression for moment of inertia of both the solid sphere and spherical shell (a) about the diameter and (b) about a tangent.

(d) Derive the expression for finding gravitational potential and field due to a solid sphere at a point (a) outside the shell (b) at the surface of the sphere (c) inside the shell.

