Total number of printed pages – 4

1A(Sem–1) MAT/ITEP

2025

MATHEMATICS

Paper : MAT0100204 – N

(Number Theory – I)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

   (a) Define greatest common divisor.

   (b) Write the value of $\tau(8)$, where τ denotes the number of positive divisors.

   (c) Define greatest integer function.

   (d) Write true or false :

   $3 \equiv 24(mod\, 7)$.

   (e) Find $\varphi(p)$, where φ is Euler's phi function and p is prime.

   (f) Define Mobius function.

   (g) What is multiplicative function ?

   (h) What is the unit digit of 7^4 ?

2. Solve the following questions : ***(any six)*** $2\times6=12$

(a) Find the value of $\sum_{n=1}^{6}\tau(n)$.

(b) Show that for $n>2, \varphi(n)$ is even integer, where $\varphi(n)$ denotes the Euler's phi function.

(c) If $a\equiv b(mod\, n)$ and $b\equiv c(mod\, n)$ then show that $a\equiv c(mod\, n)$.

(d) For any positive integer n show that $\varphi(n)=n\sum_{d|n}\frac{\mu(d)}{d}$ where $\varphi(n)$ is Euler's phi function and $\mu(n)$ is Mobius function.

(e) Show that 41 divides $2^{20}-1$.

(f) If $ca\equiv cb(mod\, n)$ then show that $a\equiv b\left(mod\frac{n}{d}\right)$, where $d=gcd(c,n)$.

(g) If f is a multiplicative function and F is defined by $F(n)=\sum_{d|n}f(d)$ then show that F is also multiplicative.

(h) Show that the cube of any integer is of the form $7k$ *or* $7k\pm1$.

(i) Prove that $7a^2-1$ is never a perfect square.

(j) If $n=q_1q_2q_3$......... where q_i's are consecutive primes then show that $n\pm1$ is a prime number.

3. Solve the following questions : ***(any four)*** $5\times4=20$

(a) If $n=p_1^{k1}p_2^{k2}\ldots\ldots.p_r^{kr}$ is a prime factorization of $n>1$ of then show that

$$\tau(n)=(k_1+1)(k_2+1)\ldots\ldots\ldots(k_r+1)$$ and

$$\sigma(n)=\frac{p_1^{k_1+1}}{p_1-1}.\frac{p_2^{k_2+1}}{p_2-1}\ldots\ldots\frac{p_r^{k_r+1}}{p_r-1}.$$

(b) Find the remainder obtained upon dividing the sum 1! + 2! + 3! +......100! by 12.

(c) Prove that no integer in the following sequences is a perfect square : 11,111,1111,11111....................

(d) Solve : $x\equiv2(mod\,3)$

$x\equiv3(mod\,5)$

$x\equiv2(mod\,7)$.

(e) For $a \geq 1$ show that the integer $a(7a^2+5)$ is of the form $7k+1$.

(f) State and prove Gauss Theorem.

(g) For $n>1$ the sum of positive integers less than and relatively prime to n is $\frac{1}{2}n\varphi(n)$.

(h) Show that $7 \mid 5^{2n}+3.2^{5n-2}$.

4. Solve the following questions : ***(any two)*** 10×2=20

(a) Show that τ and σ are multiplicative.

(b) State and prove division algorithm on number theory.

(c) State and prove Chinese Remainder Theorem.

(d) Find the last two digits in the decimal representation of 3^{256} and 11^{2025}.

(e) If p is prime then show that $(p-1)! \equiv -1(mod\, p)$.
