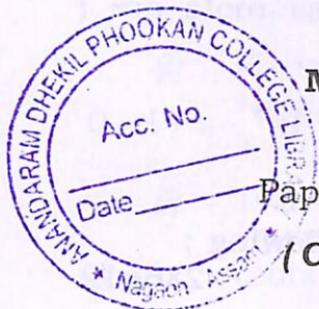


Total number of printed pages-4

1A (Sem-1/ITEP) MATO1 MN

2025



MATHEMATICS

(Minor)

Paper : MATO100104-N

(Classical Algebra)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : 1×8=8
- (a) Define symmetric matrix.
- (b) Write the expansion for $\cos x$.
- (c) Give an example of lower triangular matrix.
- (d) Fill in the blank :
 $\operatorname{sech}^2 x + \tanh^2 x = \underline{\hspace{2cm}}$
- (e) Let Z be any complex number. If Z lies on ox axis then find $\arg(Z)$.

(f) Find number of positive and negative real roots of the equation

$$x^4 + x^2 + x - 3 = 0.$$

(g) Form an equation whose roots are i and $-i$.

(h) What type of equation $4x^3 - x + 1 = 0$ is?

2. Answer **any six** from the following :

$$2 \times 6 = 12$$

(a) Show that $\cos z = \frac{e^{iz} + e^{-iz}}{2}$.

(b) Define unitary matrix with an example.

(c) Prove that

$$\cos 5\theta = 16\cos^5 \theta - 20\cos^3 \theta + 5\cos \theta.$$

(d) Express in polar form $Z = -1 - i$.

(e) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 12 & 18 \\ 4 & 9 & 2 \end{bmatrix}.$$

(f) If $A = \begin{bmatrix} 4 & 2 \\ 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 9 & 7 \end{bmatrix}$. Find $2A + 2B$.

(g) Find $\text{Log } Z$ and $\log Z$ where $Z = 1$.

(h) Solve the equation

$$x^4 - x^3 + 2x^2 - 2x + 4 = 0 \text{ whose one root is } 1 + i.$$

(i) Form an equation whose roots are reciprocals of the roots of the equation $x + 6 = 0$.

(j) Transform the general form of quadratic equation into another equation whose roots m times of the original equation.

3. Answer **any four** from the following :

$$5 \times 4 = 20$$

(a) Show that $\alpha^n + \beta^n = 2^{n+1} \cdot \cos \frac{n\pi}{3}$, where α and β are the roots of the equation $x^2 - 2x + 4 = 0$.

(b) If z_1 and z_2 are two complex numbers, show that

$$\cosh(z_1 + z_2) = \cosh z_1 \cdot \cosh z_2 + \sinh z_1 \cdot \sinh z_2$$

(c) Solve $e^z = 1 + \sqrt{3}i$.

(d) Find the fourth roots of unity.

(e) Show that

$$x^i = e^{-2n\pi} \{ \cos(\log x) - i \sin(\log x) \}.$$



(f) Solve the equation $x^3 - 3x^2 - 6x + 8 = 0$ given that the roots are in arithmetic progression.

(g) Solve the equation

$x^4 - 15x^3 + 80x^2 - 180x + 144 = 0$ given that product of one pair of roots is equal to the product of other pair.

4. Answer **any two** from the following :

$$10 \times 2 = 20$$

(a) State and prove De Moivre's theorem.

(b) Solve the following system of equations :

$$x + 3y - 2z = 0$$

$$2x + 4z - y = 0$$

$$x - 11y + 14z = 0$$

(c) (i) If "n" be any integer. Prove that

$$(1+i)^n + (1-i)^n = 2^{2^{n+1}} \cdot \cos \frac{n\pi}{4}.$$

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(ii) Express $\sin^7 \theta$ in a series of \sin of multiple of θ .

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(d) Solve $x^3 - 18x - 35 = 0$ by Cardon's method.

(e) Solve $x^4 - 3x^2 - 6x - 2 = 0$ by Euler's method.

