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1A (Sem–1/ITEP) MAT01 MN

2025

MATHEMATICS

(Minor)

Paper : MAT0100104–N

(Classical Algebra)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : 1×8=8

   (a) Define symmetric matrix.

   (b) Write the expansion for $\cos x$.

   (c) Give an example of lower triangular matrix.

   (d) Fill in the blank :

   $sec\,h^2x + tan\,h^2x =$ _____

   (e) Let Z be any complex number. If Z lies on ox axis then find $arg\,(Z)$.

(f) Find number of positive and negative real roots of the equation

$x^4 + x^2 + x - 3 = 0$.

(g) Form an equation whose roots are i and $-i$.

(h) What type of equation $4x^3 - x + 1 = 0$ is ?

2. Answer ***any six*** from the following : 2×6=12

(a) Show that $\cos z = \frac{e^{iz} + e^{-iz}}{2}$.

(b) Define unitary matrix with an example.

(c) Prove that

$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.

(d) Express in polar form $Z = -1 - i$.

(e) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 12 & 18 \\ 4 & 9 & 2 \end{bmatrix}.$$

(f) If $A = \begin{bmatrix} 4 & 2 \\ 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 9 & 7 \end{bmatrix}$. Find $2A + 2B$.

(g) Find $Log\, Z$ and $log\, Z$ where $Z = 1$.

(h) Solve the equation

$x^4 - x^3 + 2x^2 - 2x + 4 = 0$ whose one root is $1 + i$.

(i) Form an equation whose roots are reciprocals of the roots of the equation $x + 6 = 0$.

(j) Transform the general form of quadratic equation into another equation whose roots m times of the original equation.

3. Answer ***any four*** from the following : 5×4=20

(a) Show that $\alpha^n + \beta^n = 2^{n+1}.\cos\frac{n\pi}{3}$, where α and β are the roots of the equation

$x^2 - 2x + 4 = 0$.

(b) If z_1 and z_2 are two complex numbers, show that

$\cosh(z_1 + z_2) = \cos h\, z_1 . \cos h\, z_2 + \sin h\, z_1 . \sin h\, z_2$

(c) Solve $e^z = 1 + \sqrt{3}i$.

(d) Find the fourth roots of unity.

(e) Show that

$x^i = e^{-2n\pi}\{\cos(\log x) - i\sin(\log x)\}$.

(f) Solve the equation $x^3 - 3x^2 - 6x + 8 = 0$ given that the roots are in arithmetic progression.

(g) Solve the equation

$x^4 - 15x^3 + 80x^2 - 180x + 144 = 0$ given that product of one pair of roots is equal to the product of other pair.

4. Answer ***any two*** from the following : 10×2=20

(a) State and prove De Moivre's theorem.

(b) Solve the following system of equations :

$$x + 3y - 2x = 0$$
$$2x + 4z - y = 0$$
$$x - 11y + 14z = 0$$

(c) (i) If "n" be any integer. Prove that

$$(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cdot \cos\frac{n\pi}{4}.$$

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(ii) Express $\sin^7\theta$ in a series of $\sin$ of multiple of θ. 5

(d) Solve $x^3 - 18x - 35 = 0$ by Cardon's method.

(e) Solve $x^4 - 3x^2 - 6x - 2 = 0$ by Euler's method.