Total number of printed pages–7

1 (Sem–3/FYUGP) STA42MJ

2025

STATISTICS

(Major)

Paper : STA4300204 MJ

(Statistical Inference–2)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×5=5

   (a) The probability of type-I error is called ______. *(Fill in the blank)*

   (b) If t is a consistent estimator for θ then t^2 is a consistent estimator of θ^2. *(State True or False)*

   (c) Neymann Pearson Lemma provides

   (i) an unbiased test

(ii) a most powerful test

(iii) an uniformly most powerful test

(Choose the correct answer)

(d) Suppose we put forward an interval which we expect to include the true parameter value, then the process is called _____ estimation.

(Fill in the blank)

(e) If α and β represent type-I and type-II error respectively, then the power of the test is _____.

(Fill in the blank)

2. Answer ***any five*** questions from the following : 2×5=10

(a) Distinguish between estimate and an estimator.

(b) Show that if t is an unbiased estimater for θ, then t^2 is a biased estimator of θ^2.

(c) Show that in a sampling from $N(\mu, \sigma^2)$ population, then sample mean is a consistent estimator of μ.

(d) Define a Most Powerful Test.

(e) Show that if a sufficient estimator exists, it is a function of the MLE.

(f) Define a Critical Region.

(g) Describe Kolmogorov–Smirnov test for goodness of fit.

(h) Write *two* assymptotic properties of likelihood ratio test.

(i) State the invariance property of consistent estimator.

(j) Show that unbiased estimators does not always exist.

3. Answer ***any four*** questions from the following : 5×4=20

(a) Let $x_1, x_2, \cdots, x_n$ be *i.i.d.* $N(\mu, \sigma^2)$ variates.

Show that, $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator of σ^n.

(b) "An MLE may not always exist". Explain with the help of an example

(c) Let $x_1, x_2, \cdots, x_n$ be *i.i.d.* random variables with common p.d.f. .

$$P(x_i = k) = \frac{1}{N}, \quad \begin{matrix} k = 1,2,\cdots,N \\ i = 1,2,\cdots,n \end{matrix}$$

Then show that $T = x_{(n)}$ is sufficient for N.

(d) Let $x_1, x_2, \cdots, x_n$ be *i.i.d.* $b(n,p)$ variates where n and p are both unknown. Obtain their estimators using method of moments.

(e) Find the most powerful test of size α for testing $H_0 : \beta = 1$ against $H_1 : \beta = \beta_1$ based on a sample of size 1 from a population with p.d.f.

$$f(x,\beta) = \beta x^{\beta-1} \quad ,0 < x < 1$$

=0, otherwise.

(f) Let p be the probability that coin will show head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and also the power of the test.

(g) Obtain the minimum variance bound (*MVB*) estimator for μ in normal population $N(\mu, \sigma^2)$, where σ^2 is known.

(h) Define sign test. Differentiate between one sample and two sample sign test.

4. Answer ***any one*** question from the following : $10\times1=10$

(a) Define consistent estimator. State and prove the sufficient condition for consistency.

(b) (i) Obtain the most general form of the distribution differentiable in θ, for which sample mean is the M.L.E.

(ii) Let $x_1, x_2, \ldots x_n$ be a random sample from an exponential distribution with p.d.f.

$$f(x,\theta) = \theta e^{-\theta x} \quad , \; x \geq 0$$

show that $(n-1)/n\overline{x}$ is an unbiased estimator of θ.

(c) (i) With the help of an example show how Crammer Rao inequality help us to find UMVUE of a parameter.

(ii) Give the concept of distribution free method. Describe Kolmogorov–Smirnov test for two samples, explaining clearly it's purpose, assumptions and hypothesis.

(d) (i) Define a complete statistic. Let $x_1, x_2, ..., x_n$ be a sample from Bernoulli distribution

$$f(x,\theta) = \theta^x (1-\theta)^{1-x}, \quad x = 0,1$$

$$=0 \qquad , \text{elsewhere}$$

show that $\sum_{i=1}^{n} x_i$ is a complete statistic for θ.

(ii) Explain the Likelihood ratio test. Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$, where σ^2 is known. Develope a Likelihood Ratio test for testing $H_0 : \mu > \mu_0$ against $H_1 : \mu > \mu_1$.