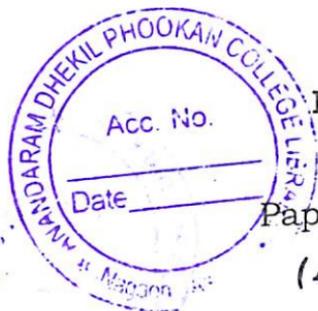


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1 (Sem-3/FYUGP) MAT42MJ

2025



MATHEMATICS

(Major)

Paper : MAT4300204 MJ

(Abstract Algebra)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

(a) Define the symmetry group of a plane figure.

(b) Under which condition the set $\{1, 2, 3, \dots, n-1\}$, $n > 1$ form a group under multiplication modulo n .

(c) Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix}$
in cyclic notation.

(d) Every permutation in S_n , $n > 1$, is a
production of _____.

(Fill in the blank)

(e) Define isomorphism between two
groups.

(f) Define $\phi: Z_{12} \rightarrow Z_{12}$ by $\phi(x) = 3x$,
 $x \in Z_{12}$. Find $\ker \phi$

(g) Give an example of a commutative ring
with unity.

(h) What is the trivial ideal of any ring R ?

2. Answer **any six** questions : $2 \times 6 = 12$

(a) Let G be a group. Show that
 $(ab)^{-1} = b^{-1}a^{-1}$, $a, b \in G$.

(b) Show that $H = \{x \in G : x \geq 1\}$ is not a
subgroup of G , where G is the group of
non-zero real numbers.

(c) Show that every cyclic group is abelian.

(d) Show that every permutation in S_n ,
 $n > 1$ is a product of 2 - cycles.

(e) Show that every subgroup of an Abelian
group is normal.

(f) Suppose ϕ is an isomorphism from
a group G to a group \bar{G} . Then prove
that, for $a \in G$, $G = \langle a \rangle$ if and only if
 $\bar{G} = \langle \phi(a) \rangle$.

(g) Let \mathbb{R}^* be the group of nonzero real
numbers under multiplication. Then
prove that, the mapping $\phi: \mathbb{R}^* \rightarrow \mathbb{R}^*$
defined by $\phi(x) = |x|$, is a
homomorphism and also find $\ker \phi$.

(h) Prove that $a(-b) = (-a)b = -ab$ for all a, b belongs to a ring \mathbb{R} .

(i) Show that $2\mathbb{Z} \cup 3\mathbb{Z}$ is not a subring of \mathbb{Z} , \mathbb{Z} the set of integers.

(j) If A and B are ideals of a ring, show that the sum $A+B = \{a+b \mid a \in A, b \in B\}$, is an ideal.

3. Answer **any four** questions : $5 \times 4 = 20$

(a) H and K be subgroups of an abelian group G . Prove that HK is a subgroup of G .

(b) Show that $U(8)$ is not a cyclic group but $U(10)$ is cyclic.

(c) Prove that the set of all even permutation in S_n , forms a subgroup of S_n .

(d) Prove that for every integer a and every prime p , $a^p \equiv a \pmod{p}$.

(e) Suppose that ϕ is an isomorphism from a group G onto a group \bar{G} . If K is a subgroup of G , then prove that $\phi(K) = \{\phi(k) \mid k \in K\}$ is a subgroup of \bar{G} .

(f) Let ϕ be a homomorphism from a group G to a group \bar{G} and let g be an element of G . If $\phi(g) = g'$, then prove that $\phi^{-1}(g') = \{x \in G \mid \phi(x) = g'\} = g \ker \phi$.

(g) Show that the set $\left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \mid a, b \in \mathbb{Z} \right\}$ of diagonal matrices is a subring of the ring of all 2×2 matrices over \mathbb{Z} .

(h) Prove that the characteristic of an integral domain is 0 or prime.

4. Answer **any two** questions : $10 \times 2 = 20$

(a) Prove that every subgroup of a cyclic group is cyclic. If $\langle a \rangle = n$, then show that the order of any subgroup of $\langle a \rangle$ is a divisor of n . $5+5=10$

(b) Let G be a group and let H be a normal subgroup of G then prove that the set $G/H = \{aH \mid a \in G\}$ is a group under the operation $(aH)(bH) = abH$.

(c) Let H and K be two normal subgroups of a group G such that $H \subseteq K$. Show that $G/K \cong G/H / K/H$.

(d) Prove that the set

$Q[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in Q\}$ is a ring.

(e) Let R be a commutative ring with unity and let A be an ideal of R . Then prove that R/A is a field if and only if A is maximal.

