Total number of printed pages–7



**1 (Sem–5/FYUGP) STA03MJ**

**2025**

# STATISTICS

( Major )

Paper : STA0500304

***( Statistical Inference–2 )***

*Full Marks : 60*

Time : 2½ hours

***The figures in the margin indicate full marks for the questions.***

1. Answer the following questions as directed : $1\times8=8$

(a) Bias of an estimator can be

(i) Positive

(ii) Negative

(iii) Either positive or negative

(iv) Always zero

*(Choose the correct option)*

(b) For Cauchy's distribution, sample mean is a consistent estimator of the population mean.

*(State True **or** False)*

(c) The correlation coefficient between a most efficient estimator and any other estimator with efficiency $e$ is ________.

*(Fill in the blank)*

(d) If a sufficient estimator exists, it is a function of the ________.

*(Fill in the blank)*

(e) Test of hypothesis $H_0 : \mu = 70$ against $H_1 : \mu > 70$ leads to :

- (i) One-sided left-tailed test
- (ii) One-sided right-tailed test
- (iii) Two-tailed test
- (iv) None of the above

(f) Power of a test is related to :

- (i) Type I error
- (ii) Type II error
- (iii) Type I and II errors both
- (iv) None of the above





(g) Type ________ error is more severe in testing of hypothesis.

*(Fill in the blank)*

(h) Non-parametric tests can be used only if the measurements are :

- (i) Nominal or ordinal
- (ii) Ratio scale
- (iii) Internal scale
- (iv) Internal and ratio scale

*(Choose the correct option)*

2. Answer ***any six*** from the following questions : 2×6=12

(a) Differentiate between estimator and estimate.

(b) If $T$ is an unbiased estimator for $\theta$, show that $T^2$ is a biased estimator for $\theta^2$.

(c) Prove that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator of $\mu$.

(d) Define type–I and type–II errors.

(e) Define most powerful test.

(f) Define simple and composite hypothesis.

(g) Find the maximum likelihood estimator of $\theta$ for $f(x, \theta) = \theta^x (1-\theta)^{1-x}; 0 \le \theta \le 1$, $x = 0$ or $1$.

(h) State the asymptotic properties of Likelihood-ratio test.

(i) Explain the test for randomness.

(j) What indication can one get from the number of runs?

3. Answer ***any four*** from the following questions : 5×4=20

(a) State and prove the invariance property of consistent estimator.

(b) Define sufficiency.
Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with p.d.f. :

$$f(x, \theta) = \theta x^{\theta-1}; 0 < x < 1, \theta > 0$$

Show that $t_1 = \prod_{i=1}^{n} X_i$, is sufficient for $\theta$. 2+3=5

(c) State the regularity conditions for Cramer-Rao inequality.





(d) Define critical region and uniformly Most Powerful test used in testing of hypothesis.

(e) Define unbiased critical region.
Given the frequency function :

$$f(x, \theta) = \begin{cases} \dfrac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{elsewhere} \end{cases}$$

and that you are testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$, by means of a single observed value of $x$. What would be the sizes of the type I and type II errors, if you choose the interval $0 \cdot 5 \le x$ as the critical region? Also obtain the power function of the test. 2+3=5

(f) Examine whether a best critical region exists for testing the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta > \theta_0$ for the parameter $\theta$ of of the distribution :

$$f(x, \theta) = \frac{1+\theta}{(x+\theta)^2}; 1 \le x < \infty$$

(g) Compare Chi-square test and Kolmogorov-Smirnov test.

(h) Describe *one* sample sign test for testing the null hypothesis that the population median is a given value.

4. Answer ***any two*** from the following questions : 10×2=20

(a) (i) A random sample $x_1, x_2, \ldots, x_n$ is taken for a normal population with mean zero and variance $\sigma^2$. Examine if $\sum_{i=1}^{n} x_i^2 / n$ is a MVB estimator for $\sigma^2$. 5



(ii) Describe the method of moments for estimating parameter. 5

(b) (i) Write the properties of Maximum Likelihood Estimator. 5

(ii) In random sampling from normal population $N(\mu, \sigma^2)$, find the maximum likelihood estimator for $\mu$ when $\sigma^2$ is known. 5

(c) (i) State and prove Neyman Pearson lemma. 5



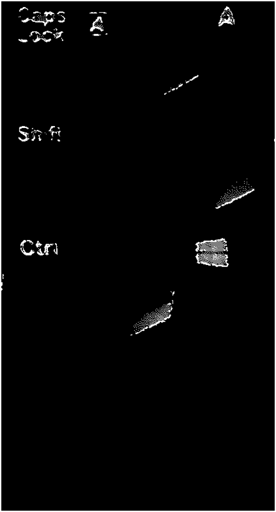
(ii) Let $p$ be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and $H_0$ is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test. 5

(d) (i) Explain briefly the likelihood ratio test. 5

(ii) Determine the best critical region for the test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 > \theta_0$ for a normal population $N(\theta, \sigma^2)$, where $\sigma^2$ is known. 5

(e) (i) State the advantages of non-parametric test. 5

(ii) Discuss Kruskal-Wallis test. 5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