Total number of printed pages–7

1 (Sem–4) STA 5

2025

STATISTICS

Paper : STA0400504

(Linear Algebra and System of Equations)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed: 1×8=8

   (a) State *one* property of Determinants.

   (b) The rank of the transpose of a matrix. is the same as that of the ______ matrix. *(Fill in the blank)*

   (c) Which of the following is standard form of Characteristic equation ?

   (i) $(A-\lambda I)=0$

   (ii) $(A-\lambda I)X=0$

   (iii) $|A-\lambda I|=0$

   (iv) $|A-\lambda I|=I$

   (Choose the correct option)

(d) The product of two determinants of same order is itself a determinant of that order. *(True **or** False)*

(e) If A is non-singular matrix, then

(i) $|A^{-1}| = 0$

(ii) $|A^{-1}| = |A|^{-1}$

(iii) $|A^{-1}| = I$

(iv) $|A| = 0$

(Choose the correct option)

(f) The intersection of *any two* subspace of a vector space is also a subspcae. *(True **or** False)*

(g) The matrix $\begin{bmatrix} 1 & 4 & 7 \\ 4 & 2 & 3 \\ 7 & -3 & 1 \end{bmatrix}$ is

(i) Symmetric matrix

(ii) Skew Symmetric matrix

(iii) Hermitian matrix

(iv) None of the above

(Choose the correct option)

(h) A subset S of a vector space is said to be a basis if S consists of linearly ______ vector. *(Fill in the blank)*

2. Answer ***any six*** questions from the following : 2×6=12

(a) Show that value of a determinant of a skew symmetric matrix of odd order is always zero.

(b) Show that every super set of a linearly dependent set of vectors is linearly dependent.

(c) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}$$

(d) Write down the system of m simultaneous non-homogeneous linear equations in n variables. Also define Augmented Matrix.

(e) Write down the quadratic forms corresponding to the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}.$$

(f) Show that the two matrices A, $C^{-1}AC$ have the same characteristic roots.

(g) Show that the three vectors (1, 2, 0), (0, 3, 1) and (–1, 0, 1) of R^3 are linearly independent.

(h) Find the inverse of the following matrix using Elementary transformations

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}.$$

(i) State the condition under which a system of non-homogeneous equations will have —

(a) No Solution

(b) Unique Solution

(j) Find the characteristic roots for the matrix A, where

$$A = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}.$$

3. Answer ***any four*** questions from the following : 5×4=20

(a) State the properties of vector space.

(b) Find the ranks of A, B, $A+B$, AB and BA where,

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}.$$

(c) Show that the vectors (1, 2, 1), (2, 1, 0) and (1, –1, 2) froms a basis of R^3.

(d) Determine the characteristic roots and corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

(e) Write down the procedure of Jacobi's iterative method to solve simultaneous linear algebraic equations.

(f) Prove that the following systems of equations are consistent

$$x - y + z = 2$$

$$3x - y + 2z = 6$$

$$3x + y + z = -18$$

(g) Solve the following equation for x

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0 .$$

(h) Solve the system of non-homogeneous linear equations

$x - 2y + 3z = 5$

$4x + 3y + 4z = 7$

$x + y - z = -4$; using Cramer rule.

4. Answer ***any two*** questions from the following : 10×2=20

(a) State and prove the Cayley-Hamilton Theorem.

(b) If x, y, z are all different and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0 ,$$

then prove that $xyz = -1$.

(c) Let Δ be the determinant of order n and D is the adjoint of Δ. Then show that $D = \Delta^{n-1}$.

(d) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix},$$

and verify that it is satisfied by A and hence obtain A^{-1}.

(e) Prove that the rank of a product of two matrices can not exceed the rank of either of the matrices.
