Total number of printed pages–7

1 (Sem–4) STA 4

2025

STATISTICS

Paper : STA0400404

(Mathematical Methods)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×8=8

(a) According to L'Hôspital's rule for indeterminate form $\frac{\infty}{\infty}$, under certain conditions imposed upon the functions $f(x)$ and $g(x)$,

if $\lim_{x\to a}\frac{f'(x)}{g'(x)}=l$, then the value of

$\lim_{x\to a}\frac{f(x)}{g(x)}$ is

(i) l

(ii) $\frac{1}{d}$

(iii) 0

(iv) None of the above

(Choose the correct option)

(b) Define beta function of first kind.

(c) If a function $f(x)$ has a maximum or minimum at a point $x = a$ within its domain, then $f'(a)$ = ______.

(Fill in the blank)

(d) Define order of a differential equation.

(e) State the necessary condition for the convergence of an infinite series ΣU_n.

(f) The value of $\Delta^2 0^3$ is

(i) 0

(ii) 1

(iii) 6

(iv) None of the above

(Choose the correct answer)

(g) A difference equation is an equation which involves—

(i) independent variable

(ii) dependent variable

(iii) the successive differences of the dependent variable

(iv) All of the above

(Choose the correct answer)

(h) The ______ interpolation formula is the average of two Gauss's formulae.

(Fill in the blank)

2. Answer ***any six*** questions from the following : 2×6=12

(a) Determine the order, degree and linearity of the following ordinary differential equation :

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 1 + x$$

(b) Evaluate $\Gamma\left(-\frac{5}{2}\right)$.

(c) If $x = r\cos\theta$, $y = r\sin\theta$,

find $\dfrac{\delta(x,y)}{\delta(r,\theta)}$.

(d) Solve the following differential equation :

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}.$$

(e) State D'Alembert's ratio test for convergence of a series.

(f) Represent the following function in factorial notation :

$$f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$$

(g) Solve the difference equation

$$U_{x+1} - 3^x U_x = 0.$$

(h) Obtain the maxima and minima of the function

$$f(x) = x^3 - 5x^2 + 8x - 4.$$

(i) Prove that $\Delta^2 x^{(m)} = m(m-1)x^{(m-2)}$, the interval of differencing being 1.

(j) Find the value of $\beta\left(\frac{2}{3}, \frac{1}{3}\right)$.

3. Answer ***any four*** questions from the following : 5×4=20

(a) Prove that $\displaystyle\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx = \beta(m,n)$.

(b) Prove that $\Gamma(n+1) = n\Gamma(n)$.

(c) If $u_1 = \dfrac{x_2 x_3}{x_1}$, $u_2 = \dfrac{x_1 x_3}{x_2}$ and $u_3 = \dfrac{x_1 x_2}{x_3}$

prove that $\dfrac{\delta(u_1, u_2, u_3)}{\delta(x_1, x_2, x_3)} = 4$.

(d) 1+4=5

(i) What do you mean by general solution of differential equation ?

(ii) Find the general solution of the following differential equation :

$$2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 24 = 0$$

(e) Solve the difference equation

$$U_{x+1} - bU_x = ca^x$$

where, c is a period function of period 1,

when (i) $b \neq a$ (ii) $b = a$.

(f) Explain the bisection method of finding the root of a polynomial equation.

(g) Show that :

$$\Delta^n 0^m = n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - \ldots.$$

and deduce that

$$n! = n^n - {}^nC_1(n-1)^n + {}^nC_2(n-2)^n - \ldots..$$

(h) Show that the series

$$1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots\ldots \text{ is convergent.}$$

4. Answer ***any two*** questions from the following : 10×2=20

(a) (i) Show that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$, to be exact is

$$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}.$$ 5

(ii) Solve the differential equation

$$(3xy - y^2)dx + x(x-y)dy = 0.$$ 5

(b) (i) Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 5

(ii) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ 5

(c) Deduce Stirling's formula for factorial n.

(d) (i) State the linear differential equation of order n with constant coefficients. 2

(ii) Solve the following differential equation : 6

$$(D^2 - 2D + 5)y = e^{-x} \quad \left[D = \frac{d}{dx}\right].$$

(iii) Obtain a differential equation from the following relation 2

$$y = A\sin x + B\cos x.$$

(e) (i) Prove that

$$2^{2m-1}\Gamma(M)\Gamma\left(M + \frac{1}{2}\right) = \sqrt{\pi}\,\Gamma(2m).$$ 6

(ii) Show that

$$\int_0^\infty e^{-ax} x^{p-1} dx = \frac{\Gamma(p)}{a^p}$$

$$a, p > 0$$ 4