Total number of printed pages-7

1 (Sem-4) STA 4

2025

Paper: STA0400404 (Mathematical Methods)

Full Marks: 60

Time: 21/2 hours

## The figures in the margin indicate full marks for the questions.

- Answer the following questions as 1. directed: 1×8=8
  - According to L'Hôspital's rule for (a) indeterminate form  $\frac{\infty}{\infty}$ , under certain conditions imposed upon the functions f(x) and g(x),

if 
$$\lim_{x \to a} \frac{f'(x)}{g'(x)} = l$$
, then the value of

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$
 is

(i) 1

(ii) 
$$\frac{1}{d}$$

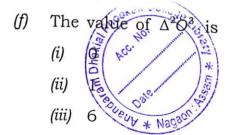
- (iii)
- (iv) None of the above

(Choose the correct option)

- Define beta function of first kind.
- If a function f(x) has a maximum or minimum at a point x = a within its domain, then f'(a) =\_\_\_\_\_.

(Fill in the blank)

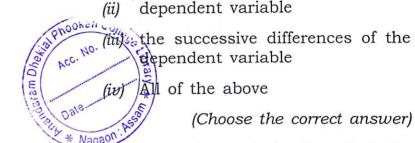
- Define order of a differential equation.
- State the necessary condition for the (e) convergence of an infinite series  $\sum U_n$ .



None of the above

(Choose the correct answer)

- A difference equation is an equation which involves
  - independent variable
  - dependent variable



\* Nagaon interpolation formula is the average of two Gauss's formulae.

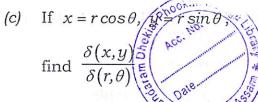
(Fill in the blank)

- Answer any six questions from the  $2 \times 6 = 12$ following:
  - Determine the order, degree and linearity of the following ordinary differential equation:

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 1 + x$$

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(b) Evaluate  $\Gamma\left(-\frac{5}{2}\right)$ .



Solve the following differential equation: (d)

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}.$$

- State D'Alembert's ratio test for (e) convergence of a series.
- Represent the following function in factorial notation:

$$f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$$

Solve the difference equation

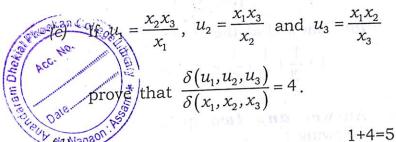
$$U_{x+1} - 3^x U_x = 0.$$

Obtain the maxima and minima of the function

$$f(x) = x^3 - 5x^2 + 8x - 4.$$

- Prove that  $\Delta^2 x^{(m)} = m(m-1)x^{(m-2)}$ , the (i) interval of differencing being 1.
- Find the value of  $\beta\left(\frac{2}{3},\frac{1}{3}\right)$ .

- Answer any four questions from the following:
  - (a) Prove that  $\int_{0}^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx = \beta(m,n).$
  - Prove that  $\Gamma(n+1) = n\Gamma(n)$ .



- What do you mean by general (i) solution of differential equation?
- Find the general solution of the following differential equation:

$$2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 24 = 0$$

Solve the difference equation

$$U_{x+1} - bU_x = ca^x$$

where, c is a period function of period 1,

when (i) 
$$b \neq a$$
 (ii)  $b = a$ .

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- Explain the bisection method of finding the root of a polynomial equation.
- (g)Show that:  $\Delta^{n} O^{m} = n^{m} - {^{n}C_{1}(n-1)^{m}} + {^{n}C_{2}(n-2)^{m}} - \dots$ and deduce that  $n! = n^n - {^nC_1(n-1)^n} + {^nC_2(n-2)^n} - \dots$
- Show that the series  $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$  is convergent.
- Answer any two questions from the following:  $10 \times 2 = 20$
- Show that the necessary and Meredicon \* Nagaon sufficient condition for the differential equation Mdx + Ndy = 0, to be exact is

$$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}.$$

- Solve the differential equation  $(3xy-y^2)dx+x(x-y)dy=0.$
- Show that  $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ .

(ii) Prove that 
$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$
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- Deduce Stirling's formula for (c) factorial n.
- State the linear differential (d) equation of order n with constant coefficients.
  - Solve the following differential equation:

$$(D^2 - 2D + 5)y = e^{-x} \qquad \left[D = \frac{d}{dx}\right].$$

- Obtain a differential equation from the following relation  $u = A \sin x + B \cos x$ .
- Prove that (e)  $2^{2m-1}\Gamma(M)\Gamma(M+\frac{1}{2}) = \sqrt{\pi}\Gamma(2m).$ 
  - Show that  $\int_{0}^{\infty} e^{-ax} x^{p-1} dx = \frac{\Gamma(p)}{a^{p}}$

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