2025

STATISTICS

Paper : STA0400304

(Probability—2 and Probability Distributions—2)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions as directed : 1×8=8

(a) If X follows normal distribution with mean 3 and standard deviation 5, find the value of mode.

(b) If X and Y are two random variables, $E(X) = E[E(X/Y)]$.

(State True or False)

(c) Under what conditions a negative binomial distribution transforms to geometric distribution?

(d) The ratio of two independent gamma variates is a beta variate of 2nd kind.

(State True or False)

(e) The standard deviation of a normal distribution is 12, find the value of quartile deviation.

(f) If X assumes only positive values and $E(X)$ and $E(1/X)$ exist, then $E(1/X) <$ equal $1/E(X)$.

(State True or False)

(g) Write down the pdf of hypergeometric distribution.

(h) If X follows exponential distributions with parameter 5. What is the value of the mean of the distributions?

2. Answer any *six* questions of the following : 2×6=12

(a) If X is a random variable which follows geometric distribution with parameter p, then find the value of $E(X)$.

(b) Define and give examples of

(i) discrete random variable;

(ii) continuous random variable.

(c) A random variable X has a mean value = 5 and variance = 3. What is the least value of $P\{|X-5|<3\}$?

(d) Write down the assumption of negative binomial distribution.

(e) If X and Y are two random variables, prove that $\text{Var}(X) = \text{Var}[E(X/Y)]$.

(f) For a normal distribution, mean = 57·9756 and 3rd Quartile = 60. Find standard deviation.

(g) Define Cauchy distribution. Mention some applications of Cauchy distribution.

(h) Prove that the moment generating function (m.g.f.) of the sum of a number of independent random variables is equal to the product of the m.g.f. of the individual variables.

(i) If X and Y be two random variables and a and b are two constants, then prove that

$$E(aX+bY) = aE(x)+bE(y)$$

(j) Can $P(S) = 2/(1+s)$ be the probability generating function of a random variable? Give reasons.

3. Answer any *four* of the following questions : 5×4=20

(a) If X_1 and X_2 be independently and identically distributed random variables,

$$P(X_i = \pm 1) = 1/2 \;;\; i = 1, 2$$

If $X_3 = X_1 X_2$, show that X_1, X_3 are independent of each other.

(b) If $\{X_n\}$ be a sequence of mutually independent random variables such that $P(X_n = \pm 2^K) = \frac{1}{2}$, examine if the law of large numbers holds good for this sequence.

(c) Obtain the distribution of

$$U = \int_{-\infty}^{x^r} f(x)\,dx$$

when it is given that x^r is the rth order statistics in an ordered sample of size n, drawn from a population having density function $dF(x) = f(x)\,dx$; $-\infty < x < \infty$.

(d) Define the beta distribution of first kind and obtain its mean and variance.

(e) If X follows $B(n, p)$ and Y follows $B(m, p)$ respectively, then prove that conditional distribution of $X/X+Y$ is hyper-geometric distribution.

(f) Give the outline of lognormal distribution and give its uses.

(g) Derive the distribution of rth order statistics in taking a random sample of size n from a continuous distribution.

(h) State and prove the weak law of large numbers.

4. Answer the following questions (any *two*) : 10×2=20

(a) Prove that gamma distribution follows normal distribution when the sample observation (n) tends to be infinite.

(b) Define negative binomial distribution. Obtain the m.g.f. and show that its mean is less than its variance.

(c) Answer the following :

(i) State and prove Chebyshev's lemma. 7

(ii) A symmetrical die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes. 3

(d) Answer the following :

(i) If X follows $N(0,1)$ and Y follows $N(0, 1)$ be independent random variables, find the distribution of X/Y. 6

(ii) Prove that a linear combination of independent normal variates is also a normal variate. 4

(e) State and prove De-Moivre's central limit theorem.

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