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3 (Sem-4/CBCS) PHY HC 1

2025

PHYSICS

(Honours Core)

Paper : PHY-HC-4016

(Mathematical Physics-III)

Full Marks : 60

Time : Three hours

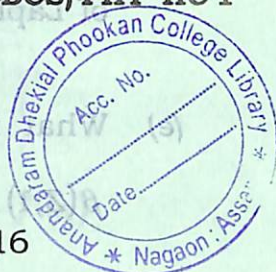
***The figures in the margin indicate
full marks for the questions.***

1. Answer the following questions : $1 \times 7 = 7$

(a) State Convolution theorem on Fourier transform.

(b) Define simple pole singularity.

(c) What is residue of a complex function ?



(d) Prove $L(1) = \frac{1}{S}$, $S > 0$, using definition of Laplace transform.

(e) What is the value of beta function $\beta(Z, 1)$?

(f) Which of the following functions is analytic everywhere ?

(i)

$$Z$$

(ii)

$$Z^{-1}$$

(iii) $\operatorname{Re} Z$

(iv) $\sin z$

(g) Evaluate $\delta_q^p A_s^{qr}$.

2. Answer the following questions: $2 \times 4 = 8$

(a) Find the Inverse Laplace transform of

$$\frac{S}{S^2 + 1}$$

(b) Prove $\int_{-\infty}^{\infty} f(t) \delta'(t-a) dt = -f'(a)$.

(c) Obtain the Fourier transform of the function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$.

(d) Write about the geometrical representation of sum of complex numbers.

3. Answer **any three** questions of the following: $5 \times 3 = 15$

(a) Using Cauchy's residue theorem,

evaluate $\int_{-\infty}^{\infty} f(x) dx$.

(b) Show that the sum function of the integral formula is $\frac{1}{2}[f(x+0)+f(x-0)]$

corresponding to the function $f(x)$ in the interval $0 < x < 1$.

(c) Find the Laplace transformation of the function $F(t) = t^n$, $n = 0, 1, 2, 3, \dots$

(d) Define symmetric and antisymmetric tensors with examples.

(e) Show that in Cartesian co-ordinate system the contravariant and covariant components of a vector are identical.

4. Answer **any three** of the following questions: $10 \times 3 = 30$

(a) (i) Express the complex number $(1+2i)/(1-3i)$ in $r(\cos\theta + i\sin\theta)$ form. 4

(ii) Using the method of residues,

show that $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi\sqrt{2}}{4}$. 6

(b) (i) Find the value of the integral

$\int_0^{1+i} (x-y-ix^2)dz$ along real axis from $z=0$ to $z=1$ and then along the line parallel to imaginary axis from $z=1$ to $z=1+i$. 5

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$ using calculus of residues. 5

(c) (i) What is Levi-Civita tensor? Prove

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^3 \sum_{j=1}^3 A_i B_j \delta_{ij} \quad 2+3=5$$

(ii) Prove Cauchy-Riemann conditions for analytical functions. 5

(d) Explain co-variant and contravariant tensors.

(e) (i) Obtain Fourier's series for the expansion of $f(x) = x \sin x$ in the

interval $-\pi < x < \pi$. Hence deduce

that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \dots$

$$6+2=8$$

(ii) Find Fourier series for $f(x)$ in the interval $(-\pi, \pi)$ where,

$$f(x) = \pi + x, \text{ when } -\pi < x < 0,$$

$$f(x) = \pi - x, \text{ when } 0 < x < \pi. \quad 2$$

(f) (i) Show that Kronecker delta is a mixed tensor of rank 2. 4

(ii) Show that the function $f(z) = \sqrt{|xy|}$ is not regular at the origin although the Cauchy-Riemann equations are satisfied at that point. 6