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3 (Sem-2/CBCS) STA HC1

2025

**STATISTICS**

(Honours Core)

Paper : STA-HC-2016

**(Probability and Probability distribution)**

Full Marks : 60

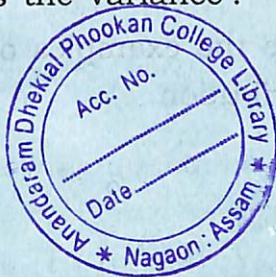
Time : Three hours

***The figures in the margin indicate  
full marks for the questions.***

1. Answer the following questions as directed :

$1 \times 7 = 7$

- (a) If  $X$  is a Binomial variate with parameters  $n$  and  $p$ , then write the value of  $\text{var}(X)$ .
- (b) The mathematical expectation of a random variable is equal to the \_\_\_\_\_ of that variable. (Fill in the blank)
- (c) If the mean of a Poisson distribution is 2, what is the variance ?
- (i) 2
- (ii) 4





(iii) 8

(iv) 5 (Choose the correct option)

(d) If the events  $A$  and  $B$  are mutually disjoint, then  $P(A \cup B) = \underline{\hspace{2cm}}$ .

(Fill in the blank)

(e) If  $A$  is a certain event, then  $P(A) = \underline{\hspace{2cm}}$

(Fill in the blank)

(f) If  $x$  and  $y$  are two  $\underline{\hspace{2cm}}$  variables then  $E(XY) = E(X) E(Y)$ .

(g) The sum of two independent gamma variates is also a gamma variate.

(State true or false)

2. Answer the following questions :  $2 \times 4 = 8$

(a) Define probability mass function and probability density function.

(b) Prove that  $E(X^2) \geq \{E(X)\}^2$

(c) Define mathematical expectation of a random variable.

(d) State the examples of Poisson distribution.

3. Answer **any three** of the following :

$$5 \times 3 = 15$$

(a) State and prove Bayes theorem.

(b) With usual notations, obtain mean and variance of binomial distribution.

(c) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

(d) Define cumulant generating function show that mean  $= k_1$ ,  $\mu_2 = k_2$ ,  $\mu_3 = k_3$ ,  $\mu_4 = k_4 + 3k_2^2$ .

(e) Obtain m.g.f of geometric distribution and hence find mean and variance.

(f) State the properties of normal distribution.

4. Answer **any three** of the following :

$$10 \times 3 = 30$$

(a) (i) State and prove the theorem of compound probability. 6

(ii) Two urns, the similar in appearance, contain the following numbers of white and black balls.



Urn I: 6 white and 4 black balls  
 Urn II: 5 white and 5 black balls.  
 One urn is selected at random and  
 a ball is drawn from it. It happens  
 to be white. What is the probability  
 that it has come from the 1st urn?

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(b) Derive the p.m.f. of negative binomial distribution. Obtain moment generating function and cumulant generating function of the distribution and hence find its mean and variance.  $4+6=10$

(c) (i) The p.d.f. of a continuous distribution is  $f(x) = y_0(x - x^2)$

$0 \leq x \leq 1$ . Find  $y_0$  and the distribution function.

5

(ii) For a binomial distribution mean = 1.0, variance = 0.8. Determine the distribution. Also find —

5

(i)  $P(X=0)$

(ii)  $P(X>0)$

(d) (i) If  $X$  is uniformly distributed with mean 1 and variance  $\frac{4}{3}$ , find  $P(X < 0)$ . Also discuss the properties of uniform distribution.

$3+3=6$

(ii) State De Moivre-Laplace and Lindeberg-Levy central limit theorem.  $2+2=4$

(e) (i) State and prove addition theorem of mathematical expectation. 5

(ii) Define characteristic function and state its important properties. 5

(f) (i) For any two events  $A$  and  $B$ , show that  
 $P(A \cap B) \leq P(A \cup B) \leq P(A) + P(B)$  4

(ii) Find mean and variance of hypergeometric distribution. 6

