Total number of printed pages – 5

3 (Sem–2/CBCS) STA HC1

2025

STATISTICS

(Honours Core)

Paper : STA–HC–2016

(Probability and Probability distribution)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1\times7=7$

   (a) If X is a Binomial variate with parameters n and p, then write the value of var (X).

   (b) The mathematical expectation of a random variable is equal to the ____ of that variable. *(Fill in the blank)*

   (c) If the mean of a Poisson distribution is 2, what is the variance ?

   (i) 2

   (ii) 4

(iii) 8

(iv) 5 *(Choose the correct option)*

(d) If the events A and B are mutually disjoint, then $P(A \cup B) =$ _______.

(Fill in the blank)

(e) If A is a certain event, then $P(A) =$ ____

(Fill in the blank)

(f) If x and y are two ______ variables then $E(XY) = E(X)\,E(Y)$.

(g) The sum of two independent gamma variates is also a gamma variate.

*(State true **or** false)*

2. Answer the following questions : 2×4=8

(a) Define probability mass function and probability density function.

(b) Prove that $E(X^2) \geq \{E(X)\}^2$

(c) Define mathematical expectation of a random variable.

(d) State the examples of Poisson distribution.

3. Answer **any three** of the following : 5×3=15

(a) State and prove Bayes theorem.

(b) With usual notations, obtain mean and variance of binomial distribution.

(c) A coin is tossed until a head appears. What is the expectation of the number of tosses required ?

(d) Define cumulant generating function show that mean $= k_1$, $\mu_2 = k_2$, $\mu_3 = k_3$, $\mu_4 = k_4 + 3k_2^2$.

(e) Obtain m.g.f of geometric distribution and hence find mean and variance.

(f) State the properties of normal distribution.

4. Answer **any three** of the following : 10×3=30

(a) (i) State and prove the theorem of compound probability. 6

(ii) Two urns, the similar in appearance, contain the following numbers of white and black balls.

Urn I : 6 white and 4 black balls
Urn II : 5 white and 5 black balls.
One urn is selected at random and a ball is drawn from it. It happens to be white. What is the probability that it has come from the 1st urn ? 4

(b) Derive the p.m.f. of negative binomial distribution. Obtain moment generating function and cumulant generating function of the distribution and hence find its mean and variance. 4+6=10

(c) *(i)* The p.d.f. of a continuous distribution is $f(x) = y_0(x - x^2)$ $0 \le x \le 1$. Find y_0 and the distribution function. 5

(ii) For a binomial distribution mean = 1.0, variance = 0.8. Determine the distribution. Also find— 5

(i) $P(X = 0)$

(ii) $P(X > 0)$

(d) *(i)* If X is uniformly distributed with mean 1 and variance $\frac{4}{3}$, find $P(X < 0)$. Also discuss the properties of uniform distribution. 3+3=6

(ii) State De Moiver-Laplace and Lindeberg-Levy central limit theorem. 2+2=4

(e) *(i)* State and prove addition theorem of mathematical expectation. 5

(ii) Define characteristic function and state its important properties. 5

(f) *(i)* For *any two* events A and B, show that

$$P(A \cap B) \le P(A \cup B) \le P(A) + P(B) \quad 4$$

(ii) Find mean and variance of hypergeometric distribution. 6
