Total number of printed pages–7

3 (Sem–3/CBCS) STA HC 1

2024

STATISTICS

(Honours Core)

Paper : STA–HC–3016

(Sampling Distributions)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×7=7

   (a) Define parameter.

   (b) The probability of type I error is called ________ . *(Fill in the blank)*

Contd.

(c) The value of test statistic which separates the critical region and the acceptance region is called the (Choose the correct option)

(i) critical value

(ii) test value

(iii) probability value

(iv) None of the above

(d) The square of a standard normal variate is called (Choose the correct option)

(i) Chi-square variate with n d.f.

(ii) t variate

(iii) Chi-square variate with 1 d.f.

(iv) F variate

(e) Write the cumulative distribution function of smallest order statistic.

(f) The null hypothesis is the hypothesis which is tested for possible rejection under the assumption that is true. (Write True **or** False)

(g) The ratio of a standard normal variate to the square root of an independent Chi-square variate divided by its degree of freedom. (Choose the correct option)

(i) Student's t

(ii) Fisher's t

(iii) F statistic

(iv) Z-test

2. Answer the following questions : 2×4=8

(a) Define sampling distribution of a statistic.

(b) Distinguish between type I and type II error.

(c) Write *two* applications of F statistic.

(d) Write *two* assumptions for student's t-test.

3. Answer ***any three*** questions from the following : 5×3=15

(a) Discuss the application of F-test in testing homogeneity of two variances.

(b) Find the cumulative distribution function of $X(n)$.

(c) Discuss different large sample tests.

(d) State and prove additive property of a Chi-square variate.

(e) If a statistic t follows student's t-distribution with n d.f., then prove that t^2 follows Snedecor's F-distribution with $(1, n)$ d.f.

Answer either 4. *(a)* **or** 4. *(b)*

4. (a) Explain the following with illustrations :

(i) Order statistic 2

(ii) One-tailed and two-tailed test 4

(iii) Standard error 2

(iv) p-value approach 2

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with continuous density. Show that $Y_1 = \min(X_1, X_2, \ldots, X_n)$ is exponential with parameter $n\lambda$ if and only if X_i is exponential with parameter λ. 10

Answer either 5. *(a)* **or** 5. *(b)*

5. (a) Derive the p.d.f. of Chi-square distribution. 10

(b) In a random and large sample, prove that

$$\chi^2 = \sum_{i=1}^{k}\left[\frac{(n_i - np_i)^2}{np_i}\right]$$

follows a Chi-square distribution appropriately with $(k-1)$ degrees of freedom, when n_i is the observed frequency and np_i is the corresponding expected frequency of the i^{th} class $(i = 1, 2, \ldots, k)$, $\sum_{i=1}^{k} n_i = n$. 10

Answer either 6. *(a)* **or** 6. *(b)*

6. (a) Let X_1 and X_2 be a random sample of size 2 from $N(0, 1)$ and Y_1 and Y_2 be a random sample of size 2 from $N(1, 1)$ and let Y_i's be independent of X_i's. Find the distribution of the following :

(i) $\dfrac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$

(ii) $\dfrac{(Y_1 + Y_2 - 2)^2}{(X_2 - X_1)^2}$ 10

(b) State and prove the relation between t, F and χ^2 distribution. 10