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**3 (Sem-3/CBCS) STA HC 1**

**2024**

**STATISTICS**

(Honours Core)

Paper : STA-HC-3016

**(Sampling Distributions)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate  
full marks for the questions.**

1. Answer the following questions as directed :  
 $1 \times 7 = 7$

(a) Define parameter.

(b) The probability of type I error is called  
\_\_\_\_\_ .  
(Fill in the blank)

Contd.

- (c) The value of test statistic which separates the critical region and the acceptance region is called the

(Choose the correct option)

- (i) critical value
- (ii) test value
- (iii) probability value
- (iv) None of the above

- (d) The square of a standard normal variate is called (Choose the correct option)

- (i) Chi-square variate with  $n$  d.f.
- (ii)  $t$  variate
- (iii) Chi-square variate with 1 d.f.
- (iv)  $F$  variate

- (e) Write the cumulative distribution function of smallest order statistic.

- (f) The null hypothesis is the hypothesis which is tested for possible rejection under the assumption that is true.

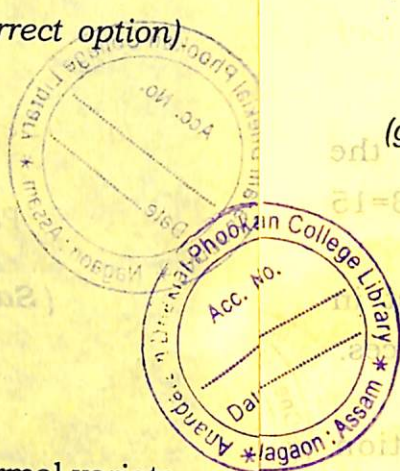
(Write True or False)

- (g) The ratio of a standard normal variate to the square root of an independent Chi-square variate divided by its degree of freedom. (Choose the correct option)

- (i) Student's  $t$
- (ii) Fisher's  $t$
- (iii)  $F$  statistic
- (iv)  $Z$ -test

2. Answer the following questions :  $2 \times 4 = 8$

- (a) Define sampling distribution of a statistic.
- (b) Distinguish between type I and type II error.





- (c) Write *two* applications of  $F$  statistic.
- (d) Write *two* assumptions for student's  $t$ -test.

3. Answer **any three** questions from the following :  $5 \times 3 = 15$

(a) Discuss the application of  $F$ -test in testing homogeneity of two variances.

(b) Find the cumulative distribution function of  $X(n)$ .

(c) Discuss different large sample tests.

(d) State and prove additive property of a Chi-square variate.

(e) If a statistic  $t$  follows student's  $t$ -distribution with  $n$  d.f., then prove that  $t^2$  follows Snedecor's  $F$ -distribution with  $(1, n)$  d.f.



Answer either 4. (a) **or** 4. (b)

4. (a) Explain the following with illustrations :

(i) Order statistic 2

(ii) One-tailed and two-tailed test 4

(iii) Standard error 2

(iv)  $p$ -value approach 2

(b) Let  $X_1, X_2, \dots, X_n$  be a random sample from a population with continuous density. Show that

$Y_1 = \min(X_1, X_2, \dots, X_n)$  is exponential with parameter  $n\lambda$  if and only if  $X_i$  is exponential with parameter  $\lambda$ . 10

Answer either 5. (a) **or** 5. (b)

5. (a) Derive the p.d.f. of Chi-square distribution. 10



- (b) In a random and large sample, prove that

$$\chi^2 = \sum_{i=1}^k \left[ \frac{(n_i - np_i)^2}{np_i} \right]$$

follows a Chi-square distribution appropriately with  $(k-1)$  degrees of freedom, when  $n_i$  is the observed frequency and  $np_i$  is the corresponding expected frequency of the  $i^{\text{th}}$  class

$$(i = 1, 2, \dots, k), \sum_{i=1}^k n_i = n. \quad 10$$

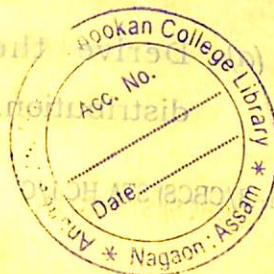
Answer either 6. (a) or 6. (b)

6. (a) Let  $X_1$  and  $X_2$  be a random sample of size 2 from  $N(0, 1)$  and  $Y_1$  and  $Y_2$  be a random sample of size 2 from  $N(1, 1)$  and let  $Y_i$ 's be independent of  $X_i$ 's. Find the distribution of the following :

(i)  $\frac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$

(ii)  $\frac{(Y_1 + Y_2 - 2)^2}{(X_2 - X_1)^2}$

10



- (b) State and prove the relation between  $t$ ,  $F$  and  $\chi^2$  distribution. 10

