3 (Sem-4/CBCS) MAT HC 2



## **MATHEMATICS**

(Honours Core)

Paper: MAT-HC-4026

(Numerical Methods)

Full Marks: 60

Time: Three hours

## The figures in the margin indicate full marks for the questions.

- Answer the following as directed:  $1 \times 7 = 7$ 1.
  - Name the three basic components of (a) an algorithm.
  - (b) Show  $\nabla E \equiv \Lambda$ .
  - Write down the Lagrangian linear (c) interpolation formula at the points  $x_0$ and  $x_1$  with corresponding function values  $f_0$  and  $f_1$ .

- (d) What is the order of convergence of secant method?
- The approximation formula for finding the derivative at  $x_0$  given by

the derivative at 
$$x_0$$
 given by 
$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \frac{h}{2} f''(\xi),$$

$$x_0 < \xi < x_{0+h}$$
is a

is a

- backward difference approximation formula of first order of approximation
- forward difference approximation formula of second order of approximation
- forward difference approximation formula of first order of approximation
- (iv) None of the above (Choose the correct option)
- What is numerical integration? What is its general form?
- Name a method for approximating a solution to an initial value problem.

- $2 \times 4 = 8$ Answer the following questions:
  - Compute the following limit and determine the rate of convergence
  - (b) Prove  $(I + \Delta)(I \nabla) \equiv I$ .
  - Show that LU decomposition of a matrix (c) is unique up to scaling by a diagonal matrix.
  - Find the approximate value of  $\int_{0}^{1} \frac{dx}{1+x}$ by Simpson's rule.
  - 5×3=15 Answer any three:
    - Construct an iteration function corresponding to the given function  $f(x) = x^3 - x^2 - 10x + 7.$

Use the fixed point iteration scheme with initial approximation as  $P_0 = 1$ and perform three iterations to approximate the root of f(x) = 0.

(b) Using the data given below form the divided difference table and use it to construct the Newton form of the interpolating polynomial:

$$x -1 0 1 2$$
  
 $y 5 1 1 11$ 

(c) Use four iterations of Newton's method to approximate the root of the equation

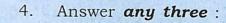
$$f(x) = x^3 + 2x^2 - 3x - 1$$

in the interval (1, 2) starting with an initial approximation of  $P_0 = 1$ .

(d) Derive the second order central difference approximation for first derivative including error term given by

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h} - \frac{h^2}{6}f'''(\xi)$$

- (e) (i) Name the measures by which errors are quantified. Write down the expressions for the same.
  - (ii) Prove that  $\Delta^n f(x_i) = (E I)^n f(x_i)$



10×3=30

- (a) What is Theoretical Error Bound? Show that the Bisection Method for approximating a root of the equation f(x) = 0 always converges. Find the order of convergence of the Bisection Method. 1+6+3=10
- (b) Verify that the equation  $x^3 + x^2 3x 3 = 0$  has a root in the interval (1, 2). Given that the exact root is  $x = \sqrt{3}$ , perform the first three iterations of the Regula-Falsi method. What is the computable estimate for  $|e_n|$ , the error obtained in nth step by this method. Verify that the absolute error in the third approximation satisfies the error estimate. 1+6+3=10
- (c) What is an interpolating polynomial? Determine the interpolation error when a function is approximated by a constant polynomial. Mention an advantage and a disadvantage of Lagrangian form of the interpolating polynomial. Derive the Lagrangian interpolating polynomial for the given data:

  1+2+2+5=10

(d) What are two different classes of methods for solving a linear system of equations. Name one method of each type. What do you mean by an LU decomposition of square matrix A.

Solve the following system using LU decomposition: 1+1+8=10

$$2x_1 + 7x_2 + 5x_3 = -4$$
$$6x_1 + 20x_2 + 10x_3 = -16$$
$$4x_1 + 3x_2 = -7$$

- (e) (i) Derive the basic Trapezoidal rule for integrating  $\int_a^b f(x)dx$ . 6
  - (ii) Use appropriate first order approximation formulas to find derivatives of the values of f(x) at the points x = 0.5, x = 0.6 and x = 0.7.

x	f(x)	f'(x)
0.5	0.4794	5
0.6	0.5646	3
0.7	0.6442	. 5



(f) What is the basic problem that is solved by Euler's method? Derive Euler's method. Given that the exact solution

to  $\frac{dx}{dt} = \frac{t}{x}$  is  $x(t) = \sqrt{t^2 + 1}$ , find the absolute error at each step that is obtained by solving

$$\frac{dx}{dt} = \frac{t}{x}$$
,  $0 \le t \le 1.0$ ,  $x(0) = 1$ ,  $h = 0.5$ 

by Euler's method. 1+4+5=10