Total number of printed pages–4

3 (Sem–2/CBCS) STA HC 2

2024

STATISTICS

(Honours Core)

Paper : STA–HC–2026

(Algebra)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×7=7

   (a) State Cayley-Hamilton theorem.

   (b) What are eigenvalues ?

   (c) “The set of all convex combinations of a finite number of points of SCR^n is a convex set.” *(State True **or** False)*

   (d) A polynomial is said to be complete if all the coefficients are present in the polynomial. *(State True **or** False)*

Contd.

(e) If any two columns of a determinant are identical, then the determinant ______. *(Fill up the blank)*

(f) Minor with proper sign is called ______. *(Fill up the blank)*

(g) The rank of a unit matrix of order n is ______. *(Fill up the blank)*

2. Answer the following questions : 2×4=8

(a) Solve the equation $x^3 - 3x^2 + 4 = 0$, given that two of its roots being equal.

(b) Prove that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent.

(c) Show that the matrices A and A' have the same eigenvalues.

(d) Prove that the characteristic roots of an orthogonal matrix are either $+1$ or -1.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Derive the standard form of a cubic equation.

(b) Show that $A.(adj\ A) = |A|.] = (adj.A)A$.

(c) Prove that the two matrices A, $P^{-1}AP$ have the same characteristic roots.

(d) Find the characteristic roots of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and verify Cayley-Hamilton theorem for this matrix.

(e) Show that

$$\begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} = (x-a)^3(x+3a)$$

4. Answer ***any three*** questions from the following : 10×3=30

(a) Show that the equations

$$\begin{aligned} x+y+z &= 6 \\ x+2y+3z &= 14 \\ x+2y+7z &= 30 \end{aligned}$$

are consistant and solve them.

(b) Show that the matrix

$$A = \begin{bmatrix} 0 & e & -b \\ -e & 0 & a \\ b & -a & 0 \end{bmatrix}$$

satisfies Cayley-Hamilton theorem.

(c) Show that if Δ is a determinant of order n, them $\Delta' = \Delta^{n-1}$.

(d) Show that the every $m \times n$ matrix of rank r can be reduced to the form $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ by a finite chain of E-operations, where I_r is the r-rowed unit matrix.

(e) Determine non-singular matrices P and Q such that PAQ is in normal form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ where $A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$

(f) Determine the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$