

Total number of printed pages-4  
3 (Sem-2/CBCS) STA HC 2

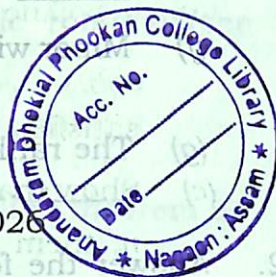
2024

**STATISTICS**

(Honours Core)

Paper : STA-HC-2026

(Algebra)



Full Marks : 60

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions as directed :  
1×7=7

(a) State Cayley-Hamilton theorem.

(b) What are eigenvalues ?

(c) "The set of all convex combinations of a finite number of points of  $\mathbb{R}^n$  is a convex set." (State True or False)

(d) A polynomial is said to be complete if all the coefficients are present in the polynomial. (State True or False)

Contd.

(e) If any two columns of a determinant are identical, then the determinant \_\_\_\_\_.  
(Fill up the blank)

(f) Minor with proper sign is called \_\_\_\_\_.  
(Fill up the blank)

(g) The rank of a unit matrix of order  $n$  is \_\_\_\_\_.  
(Fill up the blank)

2. Answer the following questions :  $2 \times 4 = 8$

(a) Solve the equation  $x^3 - 3x^2 + 4 = 0$ , given that two of its roots being equal.

(b) Prove that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent.

(c) Show that the matrices  $A$  and  $A'$  have the same eigenvalues.

(d) Prove that the characteristic roots of an orthogonal matrix are either  $+1$  or  $-1$ .

3. Answer **any three** of the following questions :  $5 \times 3 = 15$

(a) Derive the standard form of a cubic equation.

(b) Show that  $A(\text{adj } A) = |A| \cdot I = (\text{adj } A)A$ .

(c) Prove that the two matrices  $A$ ,  $P^{-1}AP$  have the same characteristic roots.

(d) Find the characteristic roots of the

$$\text{matrix } A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

and verify Cayley-Hamilton theorem for this matrix.

(e) Show that

$$\begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} = (x-a)^3(x+3a)$$

4. Answer **any three** questions from the following :  $10 \times 3 = 30$

(a) Show that the equations

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x + 2y + 7z = 30$$

are consistent and solve them.



(b) Show that the matrix

$$A = \begin{bmatrix} 0 & e-b \\ -e & 0 \\ b-a & 0 \end{bmatrix}$$

satisfies Cayley-Hamilton theorem.

(c) Show that if  $\Delta$  is a determinant of order  $n$ , then  $\Delta' = \Delta^{n-1}$ .

(d) Show that the every  $m \times n$  matrix of rank  $r$  can be reduced to the form

$\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$  by a finite chain of E-operations, where  $I_r$  is the  $r$ -rowed unit matrix.

(e) Determine non-singular matrices  $P$  and  $Q$  such that  $PAQ$  is in normal form

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \text{ where } A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$$

(f) Determine the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

