Total number of printed pages–7

3 (Sem–2/CBCS) STA HC 1

2024

STATISTICS

(Honours Core)

(Probability and Probability Distribution)

Paper : STA–HC–2016

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 7 = 7$

(a) If X is a binomial variate with parameter n and p, then write the value of $E[X]$.

(b) If A and B are two mutually exclusive events, then $P(A \cup B) = $ _____.

(Fill in the blank)

Contd.

(c) The sum of independent beta variates is also a beta variate. *(State True or False)*

(d) In the simultaneous tossing of two perfect coins, the probability of having at least one head is

(i) $\frac{1}{2}$

(ii) $\frac{1}{4}$

(iii) $\frac{3}{4}$

(iv) 1 *(Choose the correct answer)*

(e) State the conditions under which binomial distribution tends to Poisson distribution.

(f) If the mean of a gamma distribution is 3, what is its variance ?

(g) What is pairwise independent event ?

2. Answer the following questions : 2×4=8

(a) Write the probability function of uniform distribution. State its mean.

(b) With both $P(A) > 0$ and $P(B) > 0$ can two mutually exclusive events A and B be independent ? Justify your answer.

(c) For a random variable X, prove that

$$E\left[\frac{1}{X}\right] \geq \frac{1}{E[X]}$$

(d) State *two* properties of normal distribution.

3. Answer ***any three*** from the following questions : 5×3=15

(a) State and prove that Bayes' theorem.

(b) Show that for two random variables X and Y

$$E[X+Y] = E[X] + E[Y]$$

provided the expectations exist.

(c) Find the moment generating function of the normal distribution $N(\mu, \sigma^2)$.

(d) Given the joint p.d.f of two random variables X and Y

$$f(x, y) = \frac{2}{3}(x + 2y) \text{ for } \begin{matrix} 0 < x < 1 \\ 0 < y < 1 \end{matrix}$$

Find the marginal densities of X and Y.

(e) With usual notations, obtain mean and variance of Poisson distribution.

4. Answer **any three** of the following questions : 10×3=30

(a) (i) For any two events A and B, show that

$$P(A \cap B) \leq P(A \cup B) \leq P(A) + P(B)$$

5

(ii) In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40%. Of the total of their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C? 5

(b) Derive the p.m.f of negative binomial distribution. Obtain moment generating function and cumulant generating function of the distribution and hence find its mean and variance. 5+5=10

(c) (i) State the uses of log normal distribution. 2

(ii) If X has a uniform distribution in [0, 1], find the p.d.f of $-2\log X$. 3

(iii) Find the mean and variance of hypergeometric distribution. 5

(d) (i) If p.d.f of an r.v. X is given by $f(x) = 2(1-x)$ for $0 < x < 1$, then show that

$$E\left[X^r\right] = \frac{2}{(r+1)(r+2)}$$

3

(ii) Show that a linear combination of independent normal variate is also a normal variate. 4

(iii) If the r.v. X assumes the value r with probability law

$$P(x=r)=q^{r-1}p,\ r=1,2,3....$$

find the m.g.f of X. 3

(e) *(i)* If X is a gamma variate with parameter λ, obtain its m.g.f. Deduce that m.g.f of standard gamma variate tends to $\exp\left(\frac{1}{2}t^2\right)$ as $\lambda\to\alpha$. Interpret the result. 2+5=7

(ii) Define Cauchy distribution. State its one application. 2+1=3

(f) *(i)* Define conditional expectation. Prove that $E[X]=E[E(X/Y)]$ 2+3=5

(ii) If joint p.d.f

$$f(x,y)=8xy \quad 0<x<1,\ 0<y<1$$

then find

(a) $E[Y/X=x]$

(b) $E[XY/X=x]$ 5
