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3 (Sem-2/CBCS) MAT HC 2

2024

**MATHEMATICS**

(Honours Core)

Paper : MAT-HC-2026

**(Differential Equation)**

Full Marks : 60

Time : Three hours

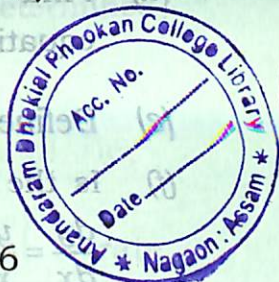
**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions : 1×7=7

(a) Find the order and degree of the following differential equation :

$$\sqrt{\left(\frac{dy}{dx}\right)^2 + 2y^2} = 4\frac{d^2y}{dx^2} + 4x$$

(b) What do you mean by explicit solution of a differential equation?



Contd.

(c) Find the Wronskian of the set  $\{x, x^2\}$ .

(d) Find complementary function of the equation  $(D^2 - 3D + 2)y = e^{5x}$ .

(e) Define integrating factor.

(f) Is the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x} \text{ homogeneous?}$$

(g) What do you mean by a singular solution of a differential equation?

2. Answer the following questions :  $2 \times 4 = 8$

(a) Show that  $e^x$  and  $e^{-x}$  are linearly independent solutions of  $\frac{d^2y}{dx^2} - y = 0$  on any interval.

(b) Find the general solution of  $y'' - 4y' + 3y = 0$ .

(c) Reduce the differential equation  $\frac{dy}{dx} + xy = x^3y^3$  in the linear form by appropriate transformation.

(d) Find the differential equation of the family of parabolas with foci at the origin and axis along the  $x$ -axis.

3. Answer the following questions : **(any three)**

$$5 \times 3 = 15$$

(a) Solve by the method of undetermined co-efficients :  $y_2 + 4y = \sin x$

(b) Solve the Euler equation  $x^2y'' + xy' - y = 0, x > 0$

(c) If a population changes at a rate proportional to its size, then population at time ' $t$ ' can be described by  $P(t) = Ae^{kt}$ .

Suppose a bacteria population grows at a rate proportional to the population. There were 200 bacteria 3 days ago, and 1000 bacteria 1 day ago. How many bacteria will there be by tomorrow?

(d) Find the complete solution of

$$\frac{d^2y}{dx^2} - \frac{3}{x} \frac{dy}{dx} + \frac{3}{x^2} y = 2x - 1$$

(e) Examine whether the differential equation  $(x^2 + y^2 + 2x)dx + 2ydy = 0$  is exact or not. Also find the integrating factor of the differential equation and solve it.

4. Answer the following questions : (any three)

$10 \times 3 = 30$

(a) Solve by method of variation of parameter

$$y'' + y = \tan x$$

(b) In a lake the pollution level is 5%. If the fresh water at the rate of 1,000 litres per day is allowed to enter and same amount of water leaves the lake, find the time when pollution level is 2.5% if volume of lake is 50,000 litres. Further, if safety level is 0.1% then after how much time water is suitable for drinking?

(c) Radium is known to decay at a rate proportional to the amount present. If the half-life of radium is 1800 years, what percentage of Radium will remain in a given sample after 900 years?

(d) Solve :  $5 + 5 = 10$

(i)  $(x^2 + 2xy^2)dx + (2x^2y + y^2)dy = 0$

(ii)  $(x^2 + y^2 + x)dx + xydy = 0$

(e) Solve :

$$(2x + y + 1)dx + (4x + 2y - 1)dy = 0$$

