Solved Examples

Unit II- Lattice vibration

Solid State Physics

1. **Compute the cut-off frequency for a linear monoatomic lattice if the velocity of sound and the interatomic spacing in the lattice are $3 \times 10^3 ms^{-1}$ and $3 \times 10^{-10} m$ respectively.**

   We know that,

   The cut-off frequency for a linear monoatomic lattice is

   $$\omega_c = \sqrt{\frac{4\beta}{m}} = \frac{2v_s}{a}$$

   Here,

   $v_s = 3 \times 10^3 ms^{-1}$

   $a = 3 \times 10^{-10} m$

   $$\omega_c = \frac{2v_s}{a} = \frac{2 \times 3 \times 10^3}{3 \times 10^{-10}} Hz = 2 \times 10^{13} Hz$$

2. **NaCl has the same structure as KCl. The Debye temperature of NaCl and KCl are 281K and 230 K respectively. If the lattice heat capacity of NaCl at 5K is $1.6 \times 10^{-2} Jmol^{-1}K^{-1}$, estiamte the heat capacity of KCl at 50K and 5K.**

   We know that for $T \ll \theta_D$, the specific heat capacity is,

   $$C_v = \frac{12}{5}\pi^4 N_a k_B \left(\frac{T}{\theta_D}\right)^3 = \frac{12}{5}\pi^4 R \left(\frac{T}{\theta_D}\right)^3$$

   Here,

   $(\theta_D)_{KCl} = 230K$

Please turn over ...

$T = 5K$

$R = 8.314 JK^{-1} mol^{-1}$

$$(C_v)_{KCl} = \frac{12}{5} \times (3.14)^4 \times 8.314 \times \left(\frac{50}{230}\right)^3 Jmol^{-1}K^{-1}$$

However, for $T << \frac{\theta_D}{10}$ above relation isn't valid. In such case, the specific heat capacity is

$$C_v = C_{ele} + C_{latt}$$

Here, $C_{ele} = aT$ is the specific heat due to free electron

And, $C_{latt} = bT^3$ is the lattice specific heat. For molecules of same structure, C_{ele} is same.

Given, $(C_v)_{NaCl,5K} = 1.6 \times 10^{-2} Jmol^{-1}K^{-1}$

$(\theta_D)_{NaCl} = 281K$ Now for $5K$ temperature,

$$(C_v)_{NaCl} = 5a + \frac{12}{5} \times (3.14)^4 \times 8.314 \times \left(\frac{5}{230}\right)^3$$

$$\implies 1.6 \times 10^{-2} = 5a + 1.09 \times 10^{-2}$$

$$\implies 5a = 0.51 \times 10^{-2}$$

$$\implies a = 0.102 \times 10^{-2}$$

Thus,

$$(C_v)_{KCl} = 5a + \frac{12}{5} \times (3.14)^4 \times 8.314 \times \left(\frac{5}{281}\right)^3$$

Or,

$$(C_v)_{KCl} = 5 \times 0.102 \times 10^{-2} + 1.9 \times 10^{-2} = 2.49 \times 10^{-2} Jmol^{-1}K^{-1}$$