Total number of printed pages–7

3 (Sem–5/CBCS) STA HC 1

2023

STATISTICS

(Honours Core)

Paper : STA–HC–5016

(Stochastic Processes and Queuing Theory)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1\times7=7$

   (a) Define a stationary process.

   (b) What is absorbing barrier ?

   (c) State *one* property of transition probability matrix.

Contd.

(d) The sum of two independent Poisson processes is also a Poisson process.

(State true or false)

(e) Mention *two* examples of stochastic process.

(f) In M/M/1 queuing model, the inter-arrival time as well as service time follows ______ distribution.

(Fill in the blank)

(g) What is the Markovian property of a stochastic process ?

2. Answer the following questions : 2×4=8

(a) State *any two* properties of Poisson process.

(b) Define bivariate probability generating function of a pair of random variables X and Y.

(c) Define stochastic matrix.

(d) State *two* characteristics of a Markov process.

3. Answer ***any three*** of the following questions : 5×3=15

(a) The transition probability matrix of a Markov chain $\{X_n; n = 1, 2, ...\}$ having three states 1, 2 and 3 is

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

and the initial distribution is

$$\pi_0 = (0.7,\ 0.2,\ 0.1)$$

Find

(i) $P_r\{X_2 = 3\}$

(ii) $P_r\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$

(b) Write a note on 'order of Markov chain'.

(c) Obtain the mean number of units in M/M/1 queuing model with finite system capacity.

(d) Let X_n be a random variable representing the weather of a particular place in a given day. Let $X_n = 0$ if the day is rainy and is equal to 1 if the day is sunny. Write the transition probability matrix. If todays weather is given what will be the weather at distant future ?

(e) What are the operating characteristics of a queuing system ?

4. Answer **either** (a) **or** (b) :

(a) (i) Write a note on graphical representation of Markov chain. 4

(ii) Find the auto-correlation coefficient between $N(t)$ and $N(t+s)$, where $\{N(t)\}$ is a Poisson process. 6

(b) (i) Consider a two-state Markov chain arising from weather condition : Cloudy (E_1) and clear (E_0), with the one-step transition probability matrix

$$P = \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}$$

What is the probability that it will be cloudy two days from now, given that it is clear to-day ? 3

(ii) Classify the following two Markov chains with the transition probabilities : 3+4=7

(i) $\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{4} & \frac{3}{4} & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{2}{4} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

5. Answer *either* (a) *or* (b) :

(a) Write a note on stochastic process explaining its applications in population studies, operation research, time series, physics and financial marketing. 10

(b) (i) Derive Chapman-Kolmogorov equation. 5

(ii) Show that the difference of two independent Poisson processes is not a Poisson process. 5

6. Answer *either* (a) *or* (b) :

(a) A self-service store employs one cashier at its counter. Nine customers arrive on an average every 5 minutes while the cashier can serve 10 customers in same time. Assuming Poisson distribution for arrival rate and exponential for service time, find :

(i) The traffic intensity. Also give its interpretation.

(ii) Average number of customers in the queue.

(iii) Average time a customer wait before being served.

(iv) Probability that cashier is idle.

(v) Probability that there are '3' customers in the system.

2+2+2+2+2=10

(b) Analyse the M/M/1/K model in detail. Also find average waiting time in the system (w) and average waiting time in the queue (w_q). 10