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3 (Sem–2/CBCS) STA HC 2

2023

STATISTICS

(Honours Core)

Paper : STA–HC–2026

(Algebra)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×7=7

(a) If two rows of a determinant are identical, the value of the determinant is _______. *(Fill in the blank)*

(b) If A be an $n \times n$ matrix with the property that $A^2 - 3A + 2I = 0$, where I is the $n \times n$ identity matrix; which of the following statements are true ?

(i) Det $(A - I) = 0$

Contd.

(ii) A is invertible

(iii) A has n distinct eigenvalues

(iv) All of the above

(Choose the correct answer)

(c) If the roots of the quadritic equation $x^2-(k+2)x+121=0$ are equal, then positive value of k is :

(i) 20

(ii) 21

(iii) 24

(iv) None of the above

(Choose the correct answer)

(d) If V_1 and V_2 are 3-dimensional subspaces of a 4-dimensional vector space V, then the smallest possible dimension of $V_1 \cap V_2$ is ______.

(Fill in the blank)

(e) For a non-singular matrix A, $(A')^{-1} = (A^{-1})'$. *(State True or False)*

(f) If A and B are symmetric matrices of same order, then $(AB' - BA')$ is a

(i) skew symmetric matrix

(ii) null matrix

(iii) symmetric matrix

(iv) None of the above

(Choose the correct option)

(g) For which value of x will the matrix given below become singular ?

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

(i) 4

(ii) 6

(iii) 8

(iv) 12

(Choose the correct option)

2. Answer the following questions : 2×4=8

(a) Define sub-space of an n-vector V_n.

(b) Solve the equation

$$3x^3 - 16x^2 + 23x - 6 = 0$$

if the product of two roots is 1.

(c) If A and B are two square matrices of order n, then show that trace (AB) = trace (BA).

(d) If A and B are n-rowed orthogonal matrices, then show that AB is also orthogonal matrix.

3. Answer **any three** of the following questions : 5×3=15

(a) Solve the equation

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

(b) If A be an $n \times n$ matrix, prove that

$$|Adj\ A| = |A|^{n-1}$$

(c) Prove that the system of equation $AX = B$ is consistant, if and only if the coefficient matrix A and the augmented matix $[A, B]$ are of the same rank.

(d) State and prove Cayley-Hamilton theorem.

(e) If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the values of

(i) $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$

(ii) $\sum \alpha^2\beta^2$

(iii) $\sum(\alpha-\beta)^2$

4. Answer the following questions : 10×3=30

(a) Write down in matrix form of the system of equations

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

and find A^{-1}, if

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

and hence solve the given equation. 10

Or

(b) (i) Show that if A is an orthogonal matrix, then A' and A^{-1} are also orthogonal. 5

(ii) Verify the orthogonality of the matrix

$$\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$$

5

(c) Find matrices P and Q so that PAQ is of the normal form, where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & -4 \\ 3 & 3 & -6 \end{bmatrix}$$

Also find the rank of A. 10

Or

(d) (i) Write the process of finding the eigenvalues and eigenvectors of a matrix. 3

(ii) Determine the eigenvalues and the corresponding eigenvectors of the matrix 7

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

(e) (i) Explain the solution of cubic equations by Cardan's method. 6

(ii) If α, β, γ are the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$, find the equation whose roots are $1+\frac{1}{\alpha}, 1+\frac{1}{\beta}, 1+\frac{1}{\gamma}$ 4

Or

(f) (i) Define vector spaces. 2

(ii) Show that the intersection of *any two* subspaces of a vector space is again a subspace of the space. 8
