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3 (Sem-2/CBCS) STA HC 2

2023

**STATISTICS**

(Honours Core)

Paper : STA-HC-2026

(Algebra)

Full Marks : 60

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions as directed :

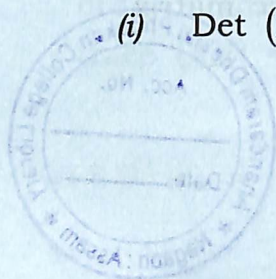
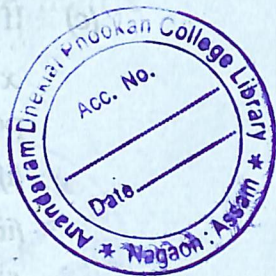
1×7=7

(a) If two rows of a determinant are identical, the value of the determinant is \_\_\_\_\_. (Fill in the blank)

(b) If  $A$  be an  $n \times n$  matrix with the property that  $A^2 - 3A + 2I = 0$ , where  $I$  is the  $n \times n$  identity matrix; which of the following statements are true ?

(i)  $\text{Det}(A - I) = 0$

Contd.





- (ii)  $A$  is invertible
- (iii)  $A$  has  $n$  distinct eigenvalues
- (iv) All of the above

(Choose the correct answer)

(c) If the roots of the quadratic equation  $x^2 - (k+2)x + 121 = 0$  are equal, then positive value of  $k$  is :

- (i) 20
- (ii) 21
- (iii) 24
- (iv) None of the above

(Choose the correct answer)

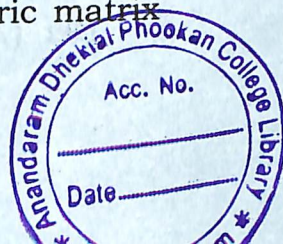
(d) If  $V_1$  and  $V_2$  are 3-dimensional subspaces of a 4-dimensional vector space  $V$ , then the smallest possible dimension of  $V_1 \cap V_2$  is \_\_\_\_\_.

(Fill in the blank)

(e) For a non-singular matrix  $A$ ,  $(A')^{-1} = (A^{-1})'$ . (State True or False)

(f) If  $A$  and  $B$  are symmetric matrices of same order, then  $(AB' - BA')$  is a

- (i) skew symmetric matrix
- (ii) null matrix



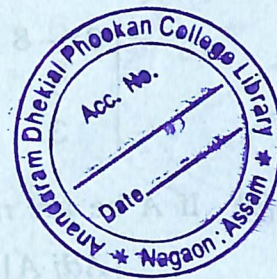
- (iii) symmetric matrix
  - (iv) None of the above
- (Choose the correct option)

(g) For which value of  $x$  will the matrix given below become singular ?

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

- (i) 4
- (ii) 6
- (iii) 8
- (iv) 12

(Choose the correct option)



2. Answer the following questions :  $2 \times 4 = 8$

(a) Define sub-space of an  $n$ -vector  $V_n$ .

(b) Solve the equation

$$3x^3 - 16x^2 + 23x - 6 = 0$$

if the product of two roots is 1.

(c) If  $A$  and  $B$  are two square matrices of order  $n$ , then show that  $\text{trace}(AB) = \text{trace}(BA)$ .



(d) If  $A$  and  $B$  are  $n$ -rowed orthogonal matrices, then show that  $AB$  is also orthogonal matrix.

3. Answer **any three** of the following questions :  $5 \times 3 = 15$

(a) Solve the equation .

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

(b) If  $A$  be an  $n \times n$  matrix, prove that  $|Adj A| = |A|^{n-1}$

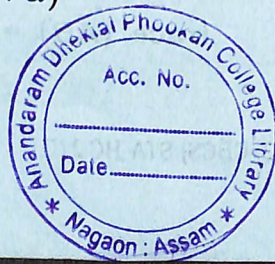
(c) Prove that the system of equation  $AX = B$  is consistent, if and only if the coefficient matrix  $A$  and the augmented matrix  $[A, B]$  are of the same rank.

(d) State and prove Cayley-Hamilton theorem.

(e) If  $\alpha, \beta$  and  $\gamma$  are the roots of the equation  $x^3 + px^2 + qx + r = 0$ , find the values of

(i)  $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha)$

(ii)  $\sum \alpha^2 \beta^2$



(iii)  $\sum (\alpha - \beta)^2$

4. Answer the following questions :  $10 \times 3 = 30$

(a) Write down in matrix form of the system of equations

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2$$

and find  $A^{-1}$ , if

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

and hence solve the given equation.

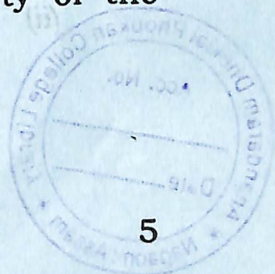
10

Or

(b) (i) Show that if  $A$  is an orthogonal matrix, then  $A'$  and  $A^{-1}$  are also orthogonal. 5

(ii) Verify the orthogonality of the matrix

$$\frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$$



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- (c) Find matrices  $P$  and  $Q$  so that  $PAQ$  is of the normal form, where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & -4 \\ 3 & 3 & -6 \end{bmatrix}$$

Also find the rank of  $A$ .

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Or

- (d) (i) Write the process of finding the eigenvalues and eigenvectors of a matrix. 3

- (ii) Determine the eigenvalues and the corresponding eigenvectors of the matrix 7

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

- (e) (i) Explain the solution of cubic equations by Cardan's method. 6

- (ii) If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + 2x^2 + 3x + 4 = 0$ , find the equation whose roots are

$$1 + \frac{1}{\alpha}, 1 + \frac{1}{\beta}, 1 + \frac{1}{\gamma} \quad 4$$

Or

- (f) (i) Define vector spaces. 2  
 (ii) Show that the intersection of *any two* subspaces of a vector space is again a subspace of the space. 8

