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3 (Sem-2/CBCS) STA HC 1

2023

STATISTICS

(Honours Core)

Paper : STA-HC-2016

(Probability and Probability Distributions)

Full Marks : 60

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer the following questions as directed :

1×7=7

(a) Define complementary event.

(b) If two events A and B are mutually exclusive, what shall be the probability of their union ?



Contd.

- (c) For any two events A and B , the probability

$$P(A|B) + P(\bar{A}|B) = \underline{\hspace{2cm}}$$

(Fill in the blank)

- (d) If X and Y are two independent random variables such that their expectation exist and $P(x \leq y) = 1$, then

(i) $E(X) \leq E(Y)$

(ii) $E(X) \geq E(Y)$

(iii) $E(X) = E(Y)$

(iv) None of the above

(Choose the correct option)

- (e) If X is a random variable with mean μ , then

$$E(X - \mu)^r$$

is called $\underline{\hspace{2cm}}$. (Fill in the blank)

Name the distribution in which probability of each successive draw varies.

- (g) The area under the standard normal curve beyond the lines $Z = \pm 1.96$ is

(i) 95 per cent

(ii) 90 per cent

(iii) 5 per cent

(iv) 10 per cent

(Choose the correct option)



2. Answer the following questions :

$$2 \times 4 = 8$$

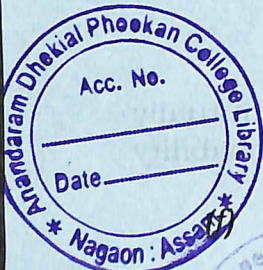
- (a) If X is a non-negative integer valued variate, then prove that

$$\sum_{K=1}^{\infty} KP(X > K) = \frac{1}{2} [E(X^2) - E(X)]$$

- (b) The distribution function F of a continuous random variable X is given by

$$F(x) = \begin{cases} 0 & , x < 0 \\ x^2 & , 0 \leq x \leq \frac{1}{2} \\ 1 - \frac{3(3-x)^2}{25} & , \frac{1}{2} \leq x < 3 \\ 1 & , x \geq 3 \end{cases}$$

Find the p.d.f. of X with comments.



(c) Define probability density function of a continuous random variable.

(d) State the important properties of distribution function.

3. Answer the following questions : **(any three)**
5×3=15

(a) Show that for two continuous random variables X and Y

$$E(X+Y) = E(X) + E(Y)$$

provided the expectations exist.

(b) For n events A_1, A_2, \dots, A_n show that

$$P\left(\bigcup_{i=1}^n A_i\right) \leq \sum_{i=1}^n P(A_i).$$

(c) State the relation between the moments and cumulants. Are the cumulants independent of change of origin and scale of the variable? Explain.

(d) For a random variable X prove that

$$E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)}$$

(e) The joint probability distribution of two random variables X and Y is given by

$$f(x, y) = 4xy e^{-(x^2+y^2)}; x \geq 0, y \geq 0$$

Find the marginal distributions and check whether X and Y are independent.

4. Answer the following questions : 10×3=30

(a) (i) State Bayes' theorem. Explain 'a priori' and 'a posteriori' probabilities in the context of this theorem. 4

(ii) Suppose that event A can occur only along the event B which in turn can occur in n mutually exclusive ways B_1, B_2, \dots, B_n . Show that

$$P(A) = \sum_{i=1}^n P(B_i) P(A|B_i)$$

(iii) If n balls are placed at a random order into n cells, find the probability that exactly one cell remains empty. 3

Or

- (b) (i) Find the m.g.f. of standard binomial variate

$$\frac{X - np}{\sqrt{npq}}$$

and obtain its limiting form as $n \rightarrow \infty$. Also interpret the result.

5+2=7

- (ii) Prove that all cumulants of the Poisson distribution are equal.

3

- (c) (i) Derive the probability mass function of negative binomial distribution.

3

- (ii) If X_1, X_2 be independent r.v.'s each having geometric distribution

$q^k p; k = 0, 1, 2, \dots$, show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform.

3

- (iii) Find the mean and variance of hypergeometric distribution.

4

Or

- (d) (i) Show that a linear combination of independent normal variates is also a normal variate. 4

- (ii) If X has a uniform distribution in $[0, 1]$, find the p.d.f. of $-2 \log X$. Also identify the distribution.

3+1=4

- (iii) Prove additive property of gamma distribution. 2

- (e) (i) If $X \sim N(\mu, \sigma^2)$, obtain the p.d.f.

of $U = \frac{1}{2} \left(\frac{X - \mu}{\sigma} \right)^2$ and identify it.

4+1=5

- (ii) If X is $\gamma(\mu)$ variate and Y is a $\gamma(\nu)$ variate then what are the distribution of

(A) $X + Y$,

(B) $\frac{X}{Y}$ and

(C) $\frac{X}{X+Y}$?

(iii) Define Cauchy distribution. 2

Or

(f) (i) If X and Y are independent with a common p.d.f. (exponential)

$$f(x) = \begin{cases} e^{-x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

find the p.d.f. of $X - Y$. 5

(ii) Find the recurrence relation for moments of normal distribution. 5

