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3 (Sem–2/CBCS) STA HC 1

2023

STATISTICS

(Honours Core)

Paper : STA–HC–2016

(Probability and Probability Distributions)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 7 = 7$

   (a) Define complementary event.

   (b) If two events *A* and *B* are mutually exclusive, what shall be the probability of their union ?

Contd.

(c) For any two events A and B, the probability

$$P(A|B)+P(\bar{A}|B)= \_\_\_\_\_\_$$

(Fill in the blank)

(d) If X and Y are two independent random variables such that their expectation exist and $P(x \le y)=1$, then

(i) $E(X) \le E(Y)$

(ii) $E(X) \ge E(Y)$

(iii) $E(X) = E(Y)$

(iv) None of the above

(Choose the correct option)

(e) If X is a random variable with mean μ, then

$$E(X-\mu)^r$$

is called ______. *(Fill in the blank)*

(f) Name the distribution in which probability of each successive draw varies.

(g) The area under the standard normal curve beyond the lines $Z = \pm 1{\cdot}96$ is

(i) 95 per cent

(ii) 90 per cent

(iii) 5 per cent

(iv) 10 per cent

(Choose the correct option)

2. Answer the following questions : 2×4=8

(a) If X is a non-negative integer valued variate, then prove that

$$\sum_{K=1}^{\infty} KP(X>K)=\frac{1}{2}\left[E(X^2)-E(X)\right]$$

(b) The distribution function F of a continuous random variable X is given by

$$\begin{aligned} F(x) &= 0 &, \quad & x<0 \\ &= x^2 &, \quad & 0 \le x \le \frac{1}{2} \\ &= 1-\frac{3(3-x)^2}{25} &, \quad & \frac{1}{2} \le x < 3 \\ &= 1 &, \quad & x \ge 3 \end{aligned}$$

Find the p.d.f. of X with comments.

(c) Define probability density function of a continuous random variable.

(d) State the important properties of distribution function.

3. Answer the following questions : ***(any three)*** 5×3=15

(a) Show that for two continuous random variables X and Y

$$E(X+Y)=E(X)+E(Y)$$

provided the expectations exist.

(b) For n events $A_1, A_2, \ldots, A_n$ show that

$$P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i).$$

(c) State the relation between the moments and cumulants. Are the cumulants independent of change of origin and scale of the variable? Explain.

(d) For a random variable X prove that

$$E\left(\frac{1}{X}\right) \ge \frac{1}{E(X)}$$

(e) The joint probability distribution of two random variables X and Y is given by

$$f(x, y) = 4xy\, e^{-\left(x^2+y^2\right)};\ x \ge 0,\ y \ge 0$$

Find the marginal distributions and check whether X and Y are independent.

4. Answer the following questions : 10×3=30

(a) (i) State Bayes' theorem. Explain 'a priori' and 'a posteriori' probabilities in the context of this theorem. 4

(ii) Suppose that event A can occur only along the event B which in turn can occur in n mutually exclusive ways $B_1, B_2, \ldots, B_n$. Show that

$$P(A)=\sum_{i=1}^{n} P(B_i)\, P(A|B_i) \quad 3$$

(iii) If n balls are placed at a random order into n cells, find the probability that exactly one cell remains empty. 3

Or

(b) (i) Find the m.g.f. of standard binomial variate

$$\frac{X-np}{\sqrt{npq}}$$

and obtain its limiting form as $n \to \infty$. Also interprete the result. 5+2=7

(ii) Prove that all cumulants of the Poisson distribution are equal. 3

(c) (i) Derive the probability mass function of negative binomial distribution. 3

(ii) If X_1, X_2 be independent r.v.'s each having geometric distribution $q^k p; k = 0, 1, 2, \ldots,$ show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform. 3

(iii) Find the mean and variance of hypergeometric distribution. 4

Or

(d) (i) Show that a linear combination of independent normal variates is also a normal variate. 4

(ii) If X has a uniform distribution in [0, 1], find the p.d.f. of $-2 \log X$. Also identify the distribution. 3+1=4

(iii) Prove additive property of gamma distribution. 2

(e) (i) If $X \sim N(\mu, \sigma^2)$, obtain the p.d.f. of $U = \frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2$ *and identify it.* 4+1=5

(ii) If X is $\gamma(\mu)$ variate and Y is a $\gamma(\nu)$ variate then what are the distribution of

(A) $X + Y$,

(B) $\frac{X}{Y}$ and

(C) $\frac{X}{X+Y}$? 3

(iii) Define Cauchy distribution. 2

Or

(f) (i) If X and Y are independent with a common p.d.f. (exponential)

$$f(x)=\begin{cases} e^{-x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

find the p.d.f. of $X-Y$. 5

(ii) Find the recurrence relation for moments of normal distribution. 5
