Total number of printed pages–7

3 (Sem–5/CBCS) STA HE 4

2022

STATISTICS

(Honours Elective)

Paper : STA–HE–5046

(Financial Statistics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : ***(any seven)*** 1×7=7

   (a) A Forward Contract is an agreement between

   (i) 2 parties

   (ii) 3 parties

   (iii) 0 parties

   (iv) many parties

   (choose the correct answer)

Contd.

(b) Defined spot price.

(c) The delivery price giving a forward contract a value of ________ is called the forward price. *(Fill in the blank)*

(d) When an investor buys an instrument he assumes

(i) short position

(ii) medium position

(iii) long position

(iv) no position

(choose the correct answer)

(e) If interest rates are constant during contract period, then forward and future price are equal. *(Write True or False)*

(f) Which distribution has importance in modelling stock prices ?

(g) Give an example of general random walk.

(h) What is put option ?

(i) What do you mean by call option ?

(j) What do you mean by straddle ?

2. Answer the following questions : ***(any four)*** 2×4=8

(a) Define intrinsic value of call option and put option.

(b) Show that :

$$E[E(Y \mid X)] = E(Y)$$

(c) Write down the assumptions of Wiener process.

(d) CRR approach to option pricing is based on the assumption of which models and how it is interpreted.

(e) Think of a strategy :

— The purchase of one stock

— The purchase of one European call with exercise price K

The sale of one European call with exercise price K

Calculate the payoff.

(f) What is mathematical expectation in financial statistics ?

(g) Define skewness and kurtosis in financial statistics.

(h) Which financial instruments are provided by the market ?

3. Answer the following questions : ***(any three)*** 5×3=15

(a) Let $X_t = \sum_{k=1}^{t} Z_k$ be a general random walk for $t = 1,2,3,....; X_0 = 0$ and $Z_1, Z_2,...$ are i.i.d. random variables with $Var(Zi) = 1; i = 1,2,...$
Calculate $Corr(X_s, X_t);\ s, t = 1,2,...$

(b) Obtain 'delta' of a European call option.

(c) The conditional variance is defined as $Var(Y \mid X) = E\left[\{Y - E(Y \mid X)\}^2 \mid X\right]$
Show that

$$Var(Y) = E\{Var(Y \mid X)\} + Var\{E(Y \mid X)\}$$

(d) Write a short note on Discrete Dividends.

(e) With usual notation, find a relationship among Δ, Γ and Θ.

(f) Suppose there is a 1-year future on a stock-index portfolio with the future price of 2,530 USD. The stock index currently is 2500, and an 2,500 USD investment in the index portfolio will pay a year-end dividend of 40 USD. Assume that the 1-year risk-free interest rate is 3%.

(i) Is this future contract mispriced ?

(ii) If there is an arbitrage opportunity, how can an investor exploit it using a zero net-investment arbitrage portfolio ?

(g) Write a note on stochastic integration.

(h) Write a note on delta hedging.

4. Answer the following questions : ***(any three)*** 10×3=30

(a) Define Wiener process. Let W_t be a standard Wiener process, then show that the process defined as

$$X_t = \frac{1}{\sqrt{c}} W_{ct};\ c > 0$$ is also a Wiener process. 2+8=10

(b) What are the assumptions of Perfect financial market ? Show that the two portfolios, which have same value at a certain time T, must have the same value at a prior time $t < T$ as well. 3+7=10

Contd.

(c) Consider the process $dS_t = \mu dt + \sigma dW_t$ where W_t is a standard Wiener process. Let $Y_t = g(S_t)$ be a process given by $Y_t = log(S_t)$. Find the SDE associated with Y_t, the mean and the variance of Y_t. 5+2+3=10

(d) Show that for a generalized binomial process with starting value X_0 and large t. The binomial distribution of the random walk X_t can be approximated by a normal distribution.

(e) Show that the price of a call option (American or European) is a convex function of the delivery price.

(f) Consider a long forward contract to buy a stock which has a price of S_t at time t. Let K be the delivery price and T be the maturity date. Further, let $V(t, S_t)$ denote the value of the long forward contract at time t and $\tau = T - t$ is the time to maturity. Assume that r is the rate of interest during the time to maturity and is constant. If the stock pays dividends at discrete time points during the time to maturity or involves any costs whose current time t discounted total value is equal to D_t, then show that

$$V(t, S_t) = V_{K,T}(t, S_t) = S_t - D_t - Ke^{-r\tau}$$

(g) Prove that, if interest rates are constant during contract period, then forward and future prices are equal.

(h) It was stated that the price of an American or European option is a convex function of its exercise price. Prove the validity of this statement for put options.
