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3 (Sem-6/CBCS) STA HC 2

2022

**STATISTICS**

(Honours)

Paper : STA-HC-6026

**(Multivariate Analysis and  
Nonparametric Analysis)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate  
full marks for the questions.**

1. Answer **any seven** of the following questions  
as directed :  $1 \times 7 = 7$

(a) The moment generating function of  
bivariate normal distribution with  
parameters  $(0, 0, \sigma_1^2, \sigma_2^2, \rho)$  is \_\_\_\_\_.

(Fill in the blank)



Contd.

(b) Let  $\underline{X} \sim N_P(\underline{\mu}, \underline{\Sigma})$ . Then the characteristic of  $\underline{X}$  is given by

(i)  $e^{it'\underline{\mu} + \frac{1}{2}t'\underline{\Sigma}t}$

(ii)  $e^{it'\underline{\mu} - \frac{1}{2}t'\underline{\Sigma}t}$

(iii)  $e^{it'\underline{\mu} + \frac{1}{2}t'\underline{\Sigma}t}$

(iv) None of the above

(Choose the correct option)

(c) Ordinary sign test considers the difference of observed values from the hypothetical median value in terms of:

(i) signs only

(ii) magnitudes only

(iii) sign and magnitude both

(iv) None of the above

(Choose the correct option)

(d) What is dispersion matrix in Multivariate data analysis?

(e) Let  $(X, Y) \sim BVND(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ .

Then state the conditional pdf of  $Y$  given  $X = x$ .

(f) What is run in non-parametric method?

(g) Define Multiple correlation coefficient.

(h) Let  $\underline{X} \sim N_3(\underline{\mu}, \underline{\Sigma})$ . Given that

$$\underline{\Sigma} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 4 \end{pmatrix}$$

Are  $X_2$  and  $X_3$  independent?

(i) The marginal distribution of a Bivariate normal distribution follows univariate normal distribution. (State True or False)

(j) The Kruskal-Wallis test is meant for:

(i) one way classification

(ii) two way classification

(iii) non classified data

(iv) None of the above

(Choose the correct option)

2. Answer **any four** of the following questions briefly: 2×4=8

(a) Define mean vector and dispersion matrix for multivariate data analysis.

(b) State the marginal pdfs of  $X$  and  $Y$  in case of  $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ .

(c) What assumptions are generally made for a non-parametric test?

(d) Let  $\underline{X} = (X_1 \ X_2 \ X_3)'$  have variance covariance matrix

$$\Sigma = \begin{pmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{pmatrix}$$

Find  $\rho_{12}$ .

(e) Define marginal distribution of  $X_1, X_2, \dots, X_k$  ( $k < p$ ) in a  $p$ -variate multivariate analysis. Also define the conditional distribution of

$X_{k+1}, X_{k+2}, \dots, X_p$  given  $X_1, X_2, \dots, X_k$ .

(f) What indication can one get from the number of runs?

(g) Give a brief idea of Principal component analysis.

(h) The pdf of bivariate normal distribution is

$$f(x, y) = k \exp \left[ -\frac{1}{2(1-\rho^2)} (x^2 - 2\rho xy + y^2) \right],$$

$$-\infty < (x, y) < \infty$$

Find the constant  $k$ .

3. Answer **any three** of the following questions: 5×3=15

(a) If  $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ , then show that  $X$  and  $Y$  are independent if and only if  $\rho = 0$ .

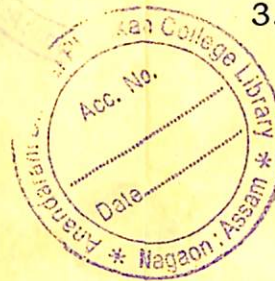
(b) Describe Kolmogorov-Smirnov one sample test stating its assumptions and hypotheses.

(c) Let  $(X, Y) \sim \text{BVND}(0, 0, 1, 1, \rho)$ . Then show that

$$Q = \frac{X^2 - 2\rho XY + Y^2}{(1-\rho^2)}$$

is distributed as chi-square with 2d.f.

(d) Let  $\underline{X} \sim N_p(\underline{\mu}, \underline{\Sigma})$ . Then find the distribution of  $C\underline{X}$  where  $C$  is a  $p \times p$  non-singular matrix of constant elements.



(e) Write an explanatory note on test of randomness.

(f) With usual notations, prove that

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

(g) Examine if Hotelling's  $T^2$  is invariant under changes in the units of measurement.

(h) Describe one sample sign test for testing the null hypothesis that the population median is a given value.

4. Answer **any three** questions from the following:  $10 \times 3 = 30$

(a) (i) State **any two** applications of multivariate analysis. 2

(ii) Let  $(X, Y) \sim \text{BVND}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ . Find the conditional distributions of  $X/Y=y$  and  $Y/X=x$ . 8

(b) Derive the probability density function of  $p$ -variate normal distribution.

(c) (i) Describe the Wilcoxon Mann-Whitney U test. 5

(ii) Let  $(X, Y) \sim \text{BVND}$  with parameters  $\mu_x = 60$ ,  $\mu_y = 75$ ,  $\sigma_x = 5$ ,  $\sigma_y = 12$  and  $\rho = 0.55$ . Then find  $P\{65 \leq X \leq 75\}$  5

(d) Let  $\underline{X}_\alpha$  ( $\alpha = 1, 2, \dots, N$ ) be a random sample from  $N_P(\underline{\mu}, \Sigma)$  and let

$$\bar{\underline{X}} = \frac{1}{N} \sum_{\alpha=1}^N \underline{X}_\alpha$$
 be the sample mean vector.

Then prove that  $\bar{\underline{X}}$  is distributed as

$$N_P\left(\underline{\mu}, \frac{\Sigma}{N}\right).$$

(e) (i) Let  $\underline{X}_\alpha^{(1)}$  ( $\alpha = 1, 2, \dots, N_1$ ) be a

random sample from  $N_P(\underline{\mu}^{(1)}, \Sigma)$

and let  $\underline{X}_\alpha^{(2)}$  ( $\alpha = 1, 2, \dots, N_2$ ) be another random sample from

$N_P(\underline{\mu}^{(2)}, \Sigma)$  where the common

dispersion matrix  $\Sigma$  is unknown. Discuss the procedure to test the

hypothesis  $H_0: \underline{\mu}^{(1)} = \underline{\mu}^{(2)}$  using

Hotelling's  $T^2$  statistic. 5

(ii) In what way the ordinary sign test can be performed for paired samples? Explain. 5



- (f) (i) State any two properties of multivariate normal distribution. 2
- (ii) Derive the bivariate normal density as a particular case of multivariate normal distribution. 8
- (g) (i) Let  $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ . Find the distribution of  $\begin{pmatrix} X_1 - X_2 \\ X_2 - X_3 \end{pmatrix}$  5
- (ii) Derive the formula for Multiple correlation coefficient for a trivariate distribution. 5
- (h) (i) Explain the distribution free method. 3
- (ii) Derive the moment generating function of a bivariate normal distribution with usual parameters. 7

