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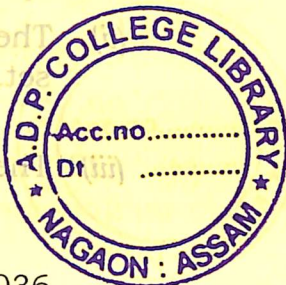
3 (Sem-3/CBCS) STA HC 3

2021

(Held in 2022)

**STATISTICS**

(Honours)



Paper : STA-HC-3036

**(Mathematical Analysis)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate  
full marks for the questions.**

1. Answer the following as directed :

1×7=7

(a) Find the infimum and supremum of

the set  $\left\{ \frac{(-1)^n}{n}; n \in N \right\}$ .

Contd. 3

(b) Identify the wrong statement :

(i) The set  $R$  of real numbers is an open set.

(ii) The set of  $Q$  of rationals is an open set.

(iii) The set  $\left\{ \frac{1}{n} : n \in N \right\}$  is not open.

(c) Show that the series  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$  is not convergent.

(d) Give the interpretation of Rolle's theorem.

(e) Suppose  $\sum u_n$  is a positive term series, such that

$$\lim_{n \rightarrow \infty} n \left( \frac{u_n}{u_{n+1}} - 1 \right) = l.$$

This series converges if

(i)  $l > 1$

(ii)  $l < 1$

(iii)  $l = 1$

(iv)  $l = 0$

(Choose the correct option)

(f) Which of the following is not correct ?

(i)  $\delta = E^{1/2} - E^{-1/2}$

(ii)  $\Delta \nabla = \Delta - \nabla$

(iii)  $\mu = \frac{1}{2} [E^{1/2} + E^{-1/2}]$

(iv)  $\Delta^2 = E^2 + 2E + 1$

(g) Which of the following is not correct ?

(i) Weddle's rule is more accurate than the Simpson's rule.

(ii) Weddle's rule requires at least seven consecutive values of  $y$ .

(iii) In Weddle's rule  $y$  is of the form

$$y = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$$

(iv) None of the above

2. Answer the following questions :  $2 \times 4 = 8$

(a) Show that the sequence  $\{S_n\}$ , where

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$
 is not convergent.

- (b) If  $M$  and  $N$  are neighbourhood of a point  $x$ , then show that  $M \cap N$  is also a neighbourhood of  $x$ .
- (c) Show that  $\sin x$  is uniformly continuous on  $[0, \infty]$ .
- (d) State the properties of divided differences.

3. Answer **any three** of the following questions :  $5 \times 3 = 15$

- (a) Show that every convergent sequence is bounded and has a unique limit.
- (b) Define positive term series. Show that the positive term geometric series  $1 + r + r^2 + \dots$  converges for  $r < 1$  and diverges to  $+\infty$  for  $r \geq 1$ .
- (c) State and prove first mean value theorem of differential calculus.
- (d) (i) Show that

$$\Delta x^m - \frac{1}{2} \Delta^2 x^m + \frac{1.3}{2.4} \Delta^3 x^m - \frac{1.3.5}{2.4.6} \Delta^4 x^m + \dots \text{ } m \text{ terms}$$

$$= \left(x + \frac{1}{2}\right)^m - \left(x - \frac{1}{2}\right)^m$$

- (ii) Define Limit superior and Limit inferior.

- (e) Prove that Newton-Gregory formula is a particular case of Newton's divided formula.

4. (a) (i) If  $\lim_{n \rightarrow \infty} a_n = l$ , then show that

$$\lim_{n \rightarrow \infty} \left( \frac{a_1 + a_2 + \dots + a_n}{n} \right) = l \quad 8$$

- (ii) Verify whether Rolle's theorem is applicable to the function

$$f(x) = 2 + (x-1)^{2/3} \text{ in the interval } [0, 2] \text{ or not.} \quad 2$$

Or

- (b) (i) Show that the sequence  $\{S_n\}$ ,

$$\text{where } S_n = \left(1 + \frac{1}{n}\right)^n \text{ is}$$

convergent and that limit

$$\left(1 + \frac{1}{n}\right)^n \text{ lies between 2 and 3.}$$

- (ii) State Cauchy's  $n$ th root test. 2
5. (a) (i) State and prove Stirling interpolation formula. 7
- (ii) Solve the difference equation  

$$y_{k+1} - ay_k = 0, a \neq 1$$
 3

Or

- (b) (i) Expand  $\sin x$  by Maclaurin's infinite series. 8
- (ii) State Taylor's theorem with Cauchy's form of remainder. 2
6. (a) (i) State and prove Weddle's rule. 7
- (ii) Show that  

$$\mu^2 y_x = y_x + \frac{1}{4} \delta^2 y_x$$
 3

Or

- (b) (i) Show that

$$\lim_{n \rightarrow \infty} \left[ \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} + \dots + \frac{1}{\sqrt{2n}} \right] = \infty$$

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- (ii) Define absolute convergence and conditional convergence.

Show that every absolutely convergent series is convergent.

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