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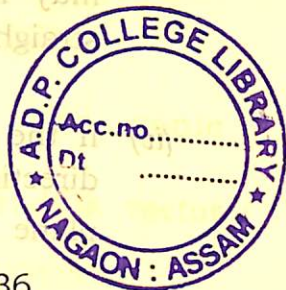
3 (Sem-3/CBCS) MAT HC 3

2021

(Held in 2022)

MATHEMATICS

(Honours)



Paper : MAT-HC-3036

(Analytical Geometry)

Full Marks : 80

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer the following questions: $1 \times 10 = 10$

(i) What is the nature of the conic represented by

$$4x^2 - 4xy + y^2 - 12x + 6y + 9 = 0 ?$$

(ii) Define skew lines.

Contd. 3

(iii) Under what condition

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
may represent a pair of parallel
straight lines ?

(iv) If the axes are rectangular, find the
direction cosines of the normal to the
plane $x + 2y - 2z = 9$.

(v) Write down the conditions under which
the general equation of second degree
 $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$
represents a sphere.

(vi) If $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is a generator of the cone
represented by the homogeneous
equation $f(x, y, z)$, then what is the
value of $f(l, m, n)$?

(vii) What is meant by diametral plane of a
conicoid ?

(viii) Find the equation of the line $\frac{x}{a} + \frac{y}{b} = 2$,
when the origin is transferred to the
point (a, b) .

(ix) Find the point on the conic
 $\frac{8}{r} = 3 - \sqrt{2} \cos \theta$ whose radius vector
is 4.

(x) What is the polar equation of a circle
when the pole is at the centre ?

2. Answer the following questions : $2 \times 5 = 10$

(a) Write down the equation to the cone
whose vertex is the origin and which
passes through the curve of intersection
of the plane $lx + my + nz = p$ and the
surface $ax^2 + by^2 + cz^2 = 1$.

(b) Transform the equation $x^2 - y^2 = a^2$ by
taking the perpendicular lines $y - x = 0$
and $y + x = 0$ as coordinate axes.

(c) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola $y^2 = 4ax$, then prove that $t_1 t_2 = -1$.

(d) Find the centre and foci of the hyperbola $x^2 - y^2 = a^2$.

(e) Find where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$ meets the plane $x+y+z=3$.

3. Answer **any four**: 5×4=20

(a) If by transformation from one set of rectangular axes to another with the same origin the expression $ax+by$ changes to $a'x'+b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$.

(b) Prove that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight

lines, if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$.

(c) Find the condition that line

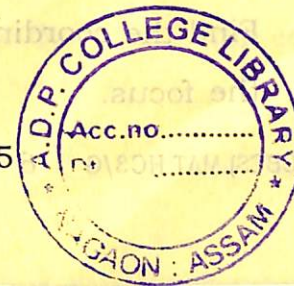
$$\frac{l}{r} = A \cos \theta + B \sin \theta$$

may touch the conic $\frac{l}{r} = 1 - e \cos \theta$.

(d) Find the equation to the plane which cuts $x^2 + 4y^2 - 5z^2 = 1$ in a conic whose centre is the point (2,3,4).

(e) Show that the equation to the cone whose vertex is origin and base is

$$z = k, f(x, y) = 0 \text{ is } f\left(\frac{kx}{z}, \frac{ky}{z}\right) = 0.$$



(f) A variable plane is at a constant distance p from the origin and meets the axes, which are rectangular in A, B, C . Through A, B, C planes are drawn parallel to the coordinate planes, show that locus of their point of intersection is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

4. Answer the following questions: $10 \times 4 = 40$

(a) Find the point of intersection of the lines represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

(b) Show that the equation $9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$ represents a parabola and it can be reduced to the standard form $Y^2 = 3X$. Find the coordinates of the vertex and the focus.

(c) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

(d) Show that the ortho-centre of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is given by $\frac{x}{l} = \frac{y}{m} = \frac{a+b}{am^2 - 2hlm + bl^2}$

(e) Find the condition that the plane $lx + my + nz = p$ may touch the conicoid $ax^2 + by^2 + cz^2 = 1$. Verify that the plane $2x - 2y + 8z = 9$ touches the ellipsoid $x^2 + 2y^2 + 3z^2 = 9$.

(f) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y + z = 0, z + x = 0, x + y = 0,$ $x + y + z = a$ is $\frac{2a}{\sqrt{6}}$ and that the three lines of shortest distance intersect at the point $x = y = z = -a$.

(g) Find the equation to the cylinder generated by the lines drawn through the points of the circle

$$x + y + z = 1, x^2 + y^2 + z^2 = 4 \text{ which are}$$

$$\text{parallel to the line } \frac{x}{2} = \frac{y}{-1} = \frac{z}{2}.$$

(h) A variable plane is parallel to the given

$$\text{plane } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0 \text{ and meets the axes}$$

in A, B, C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0.$$

