2020

STATISTICS

(Major)

Paper : 6·1

(Statistical Inference-2)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer from the given options :   1×7=7

    (a) A confidence interval of confidence coefficient $(1-\alpha)$ is considered best when it has

    (i) smallest width

    (ii) largest width

    (iii) average width

    (iv) upper and lower limits equidistant from the parameter

(b) For a certain test, you are given that $\alpha = 0.05$ and $\beta = 0.10$. The power of the test is

(i) 0.95

(ii) 0.90

(iii) 0.05

(iv) 0.10

(c) Area of the critical region depends on

(i) number of observations

(ii) value of the statistic

(iii) size of the Type I error

(iv) size of Type II error

(d) If there are 10 symbols of two types, equal in number, the minimum possible number of runs is

(i) 1

(ii) 2

(iii) 3

(iv) 5

(e) For an unbiased critical region, in testing a simple null hypothesis $H_0 : \theta = \theta_0$ (say) against a simple alternative $H_1 : \theta = \theta_1$ (say), we have,

(i) $1 - \beta < \alpha$

(ii) $1 - \beta \geq \alpha$

(iii) $\beta \geq \alpha$

(iv) $\beta + \theta_1 < \theta_0 + \alpha$

(f) Which of the following tests is equivalent / analogous to the χ^2-test of goodness of fit ?

(i) Mann-Whitney U test

(ii) Wilcoxon signed rank test

(iii) Kolmogorov-Smirnov test

(iv) Median test

(g) The ratio of the likelihood function function under H_0 and under the entire parametric space is called

   (i)   probability ratio

   (ii)   sequential probability ratio

   (iii) likelihood probability ratio

   (iv)   likelihood ratio.

2. Answer the following questions :   2×4=8

(a) Explain simple and composite hypotheses with examples.

(b) Define Uniformly Most Powerful (UMP) Critical Region and UMP test.

(c) What is the difference between sign test and Wilcoxon signed rank test ?

(d) State the asymptotic properties of Likelihood-ratio test.

3. Answer *any three* of the following questions :   5×3=15

(a) Obtain 100 $(1-\alpha)$% confidence intervals for the parameters θ and σ^2 of the normal distribution.

(b) Discuss the Kolmogorov-Smirnov two-sample test.

(c) What do you understand by a Statistical Hypothesis and a Null hypothesis ? Describe the errors involved in testing of hypothesis.

(d) State and prove the Neyman Pearson lemma.

(e) Define Spearman's rank correlation coefficient and Kendall's Tau. Discuss the similarities and differences between them.

4. Answer *any three* of the following questions :   10×3=30

(a) (i) Explain level of significance, confidence probability, power of a test and critical region.   4

(ii) Given the frequency function

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{otherwise} \end{cases}$$

For testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$, by means of a single observed value x, what would be the size of Type I and Type II errors and the Power functions of the tests for the following critical regions : 6

(a) $0 \cdot 5 \le x$ (b) $1 \le x \le 1 \cdot 5$

(b) (i) Write a note on Association and Contingency. 4

(ii) Explain the Mann Whitney U-test. To which parametric test of significance is it analogous ?

5+1=6

(c) What do you mean by Best Critical Region (BCR) ?

Using Neyman Pearson lemma, obtain the BCR for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 > \theta_0$ in case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known.

Also find the power of the test.

2+6+2=10

(d) What are the advantages of Likelihood ratio test over Neyman Pearson test ? Derive the Likelihood ratio test procedure for testing the equality of variance of two univariate normal populations. 2+8=10

(e) Define a Run. When do we use the One-sample Run test ? Describe the Wald-Wolfowitz Runs test stating the underlying assumptions, if any.

1+2+7=10

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