

2018

MATHEMATICS

( Major )

Paper : 5.5

( Probability )

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks for the questions*

1. Answer the following questions :  $1 \times 8 = 8$

(a) Write the condition for the outcomes of a random experiment so that  $p+q=1$ ;  $p$  and  $q$  being the probability of success and failure respectively.

(b) If  $S$  is the sample space in a random toss of 7 coins, then write the number of elements of  $S$ .

(c) Is the probability mass function

$x$	-1	0	1
$p(x)$	0.4	0.4	0.3

admissible? Give reason.

(d) Sketch the area under any probability curve with probability density function  $p(x)$  between  $x = c$  and  $x = d$  represented by  $P(c \leq X \leq d) = \int_c^d p(x) dx$ .

(e) For a discrete random variable  $X$  with probability function  $p(x)$ ,  $r$ th moment about  $A$  is  $\sum (x - A)^r p(x)$ . What are the values of  $r$  and  $A$  for (i)  $E(X)$  and (ii)  $\text{var}(X)$ ?

(f) The density function of a random variable  $X$  is given by

$$f(x) = \begin{cases} 2, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find  $E(X)$ .

(g) What is the probability of getting exactly 3 heads in 6 tosses of a fair coin?

(h) Name the distribution in which mean is the square of its standard deviation.

2. Answer any four of the following :  $3 \times 4 = 12$

(a) If  $A$  and  $B$  are two possible outcomes of an experiment and  $p(A) = 0.4$ ,  $P(A \cup B) = 0.7$  and  $P(B) = p$ , then for what value of  $p$ ,  $A$  and  $B$  become independent?

(b) A random variable  $X$  has the following probability function :

$x$	0	1	2	3	4	5	6	7
$p(x)$	0	$k$	$2k$	$2k$	$3k$	$k^2$	$2k^2$	$7k^2 + k$

Evaluate  $P(X \geq 6)$  and  $P(0 < x < 5)$ .

(c) Show that in a frequency distribution

$$(x_i, f_i); i = 1, 2, \dots, n$$

mathematical expectation of the random variable is nothing but its arithmetic mean.

(d) Define Poisson distribution and

hence prove that  $\sum_{r=0}^{\infty} p(r) = 1$ .

(e) If the random variable  $X$  is normally distributed with mean  $\mu$  and variance  $\sigma^2$ , show that the mean of the variate

$$z = \frac{x - \mu}{\sigma}$$

is always zero.

3. Answer any *two* from the following :  $5 \times 2 = 10$

(a) Prove that two events  $A$  and  $B$  are independent  $\Leftrightarrow P(A \cap B) = P(A) P(B)$ .

(b) A man has five coins, one of which has two heads. He randomly takes out a coin and tosses in three times. What is the probability that it will fall head upward all the times?

(c) For two independent events  $A$  and  $B$ , prove that (i)  $A$  and  $\bar{B}$  are independent and (ii)  $\bar{A}$  and  $\bar{B}$  are independent.

4. Answer any *two* from the following :

5×2=10

- (a) Let  $X$  and  $Y$  be two random variables each taking three values  $-1, 0, 1$  and having the joint probability distribution as given in the following table :

$X \backslash Y$	$-1$	$0$	$1$
$-1$	$0$	$\cdot 1$	$\cdot 1$
$0$	$\cdot 2$	$\cdot 2$	$\cdot 2$
$1$	$0$	$\cdot 1$	$\cdot 1$

Obtain the marginal probability distribution of  $X$  and  $Y$ .

- (b) The probability function of a random variable  $X$  is given by

$$f(x) = \begin{cases} x^2 / 81, & -3 < x < 6 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability density function for the random variable

$$u = \frac{1}{3}(12 - X)$$

(c) A random variable  $X$  has density function

$$f(x) = \begin{cases} ce^{-3x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Find (i) the constant  $c$ , (ii)  $P(1 < x < 2)$  and (iii)  $P(X \geq 3)$ .

5. Answer any *two* from the following :  $5 \times 2 = 10$

(a) Prove that

$$\text{var}(ax + by) = a^2 \text{var}(x) + b^2 \text{var}(y) + 2ab \text{cov}(x, y)$$

(b) A random variable has the following probability distribution :

$x$	0	1	2	3
$p(x)$	0.1	0.3	0.4	0.2

Find (i)  $E(X)$  and (ii)  $\text{var}(X)$ .

(c) A continuous random variable  $X$  has the probability function given by

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Find (i)  $E(X)$  and (ii)  $E(X^2)$ .

6. Answer any *two* from the following :  $5 \times 2 = 10$

(a) Prove that for the binomial distribution with parameter  $n$  and  $p$ , variance cannot exceed  $\frac{n}{4}$ .

(b) Derive Poisson distribution as a limiting case of binomial distribution.

(c) Prove that the mean and variance of a binomially distributed variable are respectively  $\mu = np$  and  $\sigma^2 = npq$ .

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