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3 (Sem-6) MAT M 2

2020

MATHEMATICS

(Major)

Paper : 6·2

(Numerical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) If $\pi=\frac{22}{7}$ is approximated as 3·14, find the relative error and relative percentage error.

   (b) Define 'absolute error'.

   (c) Find the difference $\sqrt{2{\cdot}01}-\sqrt{2}$, correct to three significant figures.

Contd.

(d) If m and n are positive integers, then show that $\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$.

(e) Evaluate $\Delta^n\left(\frac{1}{x}\right)$, with 1 as the interval of differencing.

(f) Give the relationship between the operator Δ and the differential operator D.

(g) Write the general quadrature formula in numerical integration.

2. Answer the following questions : 2×4=8

(a) Find the number of significant figures in $x = 0{\cdot}3941$ whose absolute error is $0{\cdot}25 \times 10^{-2}$.

(b) Given $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$ and $u_5 = 8$, find $\Delta^5 u_0$.

(c) What is numerical differentiation ? Explain briefly its importance.

(d) Derive trapezoidal rule from Newton-Cotes quadrature formula.

3. Answer the following questions : 5×3=15

(a) Find the relative error for evaluation of $u = x_1 x_2$ with $x_1 = 4{\cdot}51$, $x_2 = 8{\cdot}32$ having absolute errors $\Delta x_1 = 0{\cdot}01$ in x_1 and $\Delta x_2 = 0{\cdot}01$ in x_2.

(b) Using the method of separation of symbols, prove the following :

$$(u_1 - u_0) - x(u_2 - u_1) + x^2(u_3 - u_2) - \ldots\ldots$$

$$= \frac{\Delta u_0}{1+x} - x\frac{\Delta^2 u_0}{(1+x)^2} + x^2\frac{\Delta^3 u_0}{(1+x)^3} - \ldots.$$

Or

Find the function whose first difference is $9x^2 + 11x + 5$.

(c) A second degree polynomial passes through the points (1, –1), (2, –1), (3, 1) and (4, 5). Find the polynomial.

Or

Using Lagrange's interpolation formula, find the form of the function given by :

x	3	2	1	–1
$f(x)$	3	12	15	–21

4. Answer ***any one*** part :

(a) *(i)* Apply Stirling's formula to find a polynomial of degree 4 which takes the following tabular values :

x :	1	2	3	4	5
$y=f(x)$:	1	-1	1	-1	1

(ii) Using Newton's divided difference formula, construct the interpolating polynomial and hence compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=5$ using the following data :

x :	0	2	3	4	7	9
y :	4	26	58	112	466	922

5+5=10

(b) *(i)* Use Bessel's formula to find $y(0{\cdot}12)$ from the following data :

x :	0	0·05	0·1	0·15	0·2	0·25
y :	0	0·10017	0·20134	0·30452	0·41075	0·52110

(ii) Find the value of $\int_1^5 \log_{10} x\,dx$, taking 8 subintervals, by trapezoidal rule. 5+5=10

5. Answer ***any one*** part :

(a) *(i)* In a machine a slider moves along a fixed straight rod. Its distance *x cms* along the rod is given below for various values of time *t* seconds. Find the velocity and acceleration of the slider when $t=0{\cdot}3$.

$t(sec)$:	0	0·1	0·2	0·3	0·4	0·5	0·6
$x(cm)$:	30·13	31·62	32·87	33·64	33·95	33·81	33·24

(ii) The velocity v *(km/min)* of a car which starts from rest, is given at fixed intervals of time t *(min)* as follows :

t :	2	4	6	8	10	12	14	16	18	20
v :	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes. 5+5=10

(b) (i) Using Lagrange's formula and the following table, find $f'(3)$ and $f'(4)$:

x	1	2	4	8	10
$f(x)$	0	1	5	21	27

(ii) Find an approximate value of $\log_e 7$ using Simpson's rule to the integral $\int_1^7 \frac{dx}{x}$.

5+5=10

6. Answer ***any one*** part :

(a) (i) Derive the rate of convergence of the Secant method.

(ii) Compute the root of $e^x - 3x = 0$, using bisection method, lying between 1·5 and 1·6, correct to two decimal places.

5+5=10

(b) (i) Using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$, which is nearer to $x=2$, correct to three decimal places.

(ii) Find an approximate root of the equation $x^3 + x - 1 = 0$ near $x = 1$, by the Regula-Falsi method, correct to two decimal places.

5+5=10