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3 (Sem 2) MAT M2

2015

**MATHEMATICS**

**(Major)**

Theory Paper : 2.2

**(Differential Equation)**

Full Marks - 80

Time - Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed :  $1 \times 10 = 10$

(a) Give a definition of an orthogonal trajectory of a family of curves.

(b) What is an exact differential equation ?

(c) When a first order differential equation is said to be homogeneous ?

(d) Define a linear differential equation.

[Turn over

- (e) Write down the complementary function of the differential equation  $(D^2 + 1)y = e^{2x}$ .
- (f) Is the integrating factor of a differential equation  $Mdx + Ndy = 0$  unique ?
- (g) What is the geometrical interpretation of a differential equation ?
- (h) The slope of a plane curve at a point is proportional to the product of the coordinates of that point. Express it by a differential equation.
- (i) Write the form of a total differential equation
- (j) Write the particular integral of the differential equation  $(D^2 - 3D + 2)y = e^x$ .

2. Answer the following questions :  $2 \times 5 = 10$

(a) Solve :  $x^2(y-1)dx + y^2(x+1)dy = 0$

(b) Find the differential equation of

$y = e^x (a \cos x + b \sin x)$  where  $a$  and  $b$  are constants.

(c) Solve :  $\frac{dy}{dx} + \frac{\sin 2y}{x} = x^3 \cos^2 y$

(d) Find the integrating factor of

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^2$$

(e) Construct the partial differential equation by eliminating  $a, b, c$  from  $z = a(x+y) + b(x-y) + abt + c$ .

3. Answer any *five* questions : 4×5=20

(a) Solve :  $x dx + y dy + \left(1 + \frac{y^2}{x^2}\right) (y dx - x dy) = 0$ .

(b) Find the orthogonal trajectories of the family of co-axial circles

$$x^2 + y^2 + 2gx + c = 0 \text{ where } g \text{ is the parameter.}$$

(c) Solve :  $(1 + y^2) dx = (\tan^{-1} y - x) dy$

(d) Apply the method of variation of parameters to solve

$$\frac{d^2 y}{dx^2} + 4y = 4 \tan 2x$$

(e) Solve :  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$ , given that

$x + \frac{1}{x}$  is one of its solutions.

(f) Solve :  $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2 - y^2)}$

(g) Solve :  $(D^3 + 1)y = \cos x$

4. Answer either (a) and (b) or (c) and (d) :

5+5=10

(a) Find the complete integral of  $p^2x + q^2y = z$

(b) Solve :  $y = px + \sqrt{a^2p^2 + b^2}$

(c) Solve by Charpit's method

$$z = px + qy + p^2 + q^2$$

(d) Solve :  $(D^2 - 2D + 1)y = x^2 e^{3x}$

5. Find the condition of exactness of a linear differential equation of order  $n$ . 6

Or

Find the necessary condition for integrability of the total differential equation  $Pdx + Qdy + Rdz = 0$ .

6. Answer either (a) and (b) or (c) and (d) :

$$6+6=12$$

(a) Find  $f(y)$  such that  $f(y)dx - zx dy - xy dz = 0$  is integrable. Hence solve it.

(b) Eliminate the arbitrary function  $\phi$  from the equation  $\phi(x + y + z, x^2 + y^2 + z^2) = 0$ .

What is the order of this differential equation ?

(c) Solve :  $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$

(d) Verify that the condition of integrability is satisfied by the equation  $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$  and solve it.

7. Answer any two questions :  $6 \times 2 = 12$

(a) Solve :  $\frac{d^2y}{dx^2} - 2 \tan x \frac{dy}{dx} + 3y = 2 \sec x$

given that  $y = \sin x$  is a part of its complementary function.

(b) Find the integral surface of the linear partial differential equation  $(x-y)p + (y-x-z)q = z$  through the circle  $z = 1, x^2 + y^2 = 1$ .

(c) Reduce the differential equation  $(px-y)(x-py) = 2p$  to Clairaut's form by the substitution  $x^2 = u$  and  $y^2 = v$  and find its complete primitive and its singular solution, if any.