2017

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatories)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) If A and B are two sets and A' denotes the complement of A, then $A \cap (A \cup B)'$ is equal to

(i) A

(ii) B

(iii) ϕ

(iv) $A \cap B$

(b) In a class, there are 8 boys and 6 girls. The teacher wants to select either a boy or a girl to represent the class in a function. In how many ways the teacher can perform this job?

(c) How many edges and vertices are there in the graph of $K_{2,5}$?

(d) Draw the graph of K_5.

(e) Under what condition of n, the complete graph K_n is Eulerian?

(f) If $r(m, n)$ is a Ramsay number, find $r(2, 2)$.

(g) Let G be a graph with n edges and m vertices, then find

$$\deg(v_1) + \deg(v_2) + \cdots + \deg(v_m)$$

2. Answer the following questions : 2×4=8

(a) In a connected graph, show that number of odd degree vertices is even.

(b) Let H be a subgraph of a graph G and $\overline{H}$ be the complement of H in G. Draw the graphs H and $\overline{H}$ from the graph G.

(c) State the hand-shaking theorem. What is meant by source and sink in graph theory?

(d) State only two properties of tree.

3. Answer any *three* of the following questions : 5×3=15

(a) For any natural number n and $o < r \leq n$, prove that

$$P(n, r) = \frac{n!}{(n-r)!}$$ 5

(b) Suppose $S = \{1, 2, 3, ..., n\}$, where $n \geq 1$. Find the number of ordered pairs (A, B) of subsets of S such that

(i) $A \neq B$

(ii) $A \cup B = S$ 2+3=5

(c) If $G = (V, E)$ is a (p, q) graph, then show that

$$\delta(G) \leq \frac{2q}{p} \leq \Delta(G)$$

where $\delta(G) = \min\{\deg(v_i) : v_i \in V\}$ and $\Delta(G) = \max\{\deg(v_i) : v_i \in V\}$. 5

(d) What do you mean by saying a connected graph? If G is not a connected graph, then show that $\overline{G}$ is connected. 1+4=5

(e) Prove that the edge connectivity of a connected graph cannot exceed the minimum degree of G. 5

4. *(a)* Enumerate the number of non-negative integral solutions to the inequality

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 19$$ 4

(b) *(i)* What is combinatorial identity? Also find the number of 3 combinations of

$$\{\infty \cdot a_1, \infty \cdot a_2, \infty \cdot a_3, \infty \cdot a_4\}$$

(ii) Find the number of binary numbers with five 1s and three 0s.

(iii) In how many ways can a king or a queen be drawn from an ordinary deck of playing cards? 2+2+2=6

5. *(a)* If G is a simple graph with n vertices and k components, then show that G can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 6

(b) Prove that every non-trivial tree has at least 2 vertices of degree 1. 4

Or

Prove that a vertex v in a connected graph G is a cut vertex if and only if there exist vertices u and w distinct from v such that every path connecting u and w contains the vertex v.

6. *(a)* Define Hamiltonian path and Hamiltonian circuit with an example for each. 4

(b) Give an example of a graph that contains—

(i) an Eulerian circuit that is also a Hamiltonian circuit;

(ii) neither an Eulerian circuit nor a Hamiltonian circuit. 1+1=2

(c) Let G be a connected graph which contains an Euler trail but not Euler circuit, then show that G has exactly two vertices of odd degree. 4

Or

When is a graph called Hamiltonian graph? Also show that every Hamiltonian graph is 2-connected.

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