

2017

MATHEMATICS

(Major)

Paper : 6.4

(Discrete Mathematics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks
for the questions

1. Answer the following as directed : 1×7=7

(a) Write the fundamental theorem of arithmetic.

(b) If the positive integers a and b are relatively prime, then

(i) $[a, b] = 1$

(ii) $a|b$

(iii) $(a, b) = 1$

(iv) None of the above

(Choose the correct option)

(c) If a and b are two positive integers such that $(a, b) = 1$, then

(i) $[a, b] = 1$

(ii) $[a, b] = ab$

(iii) $(a, b) [a, b] = 1$

(iv) None of the above

(Choose the correct option)

(d) State Wilson's theorem on congruence.

(e) State Fermat's last theorem.

(f) If $d = (a, b)$, $d|c$ and $ax + by = c$ has a particular solution x_0 and y_0 , then write the other solution of the equation.

(g) Express 113 as a sum of two squares.

2. Answer the following questions : 2×4=8

(a) If $a|bc$ and $(a, b) = 1$, then show that $a|c$.

(b) What is the remainder when 5^{48} is divided by 12?

(c) Solve the linear congruence equation

$$6x \equiv 15 \pmod{21}$$

(d) Prove that the equation $x^4 + y^4 = z^4$ has no positive solution.

3. Answer the following questions : $5 \times 3 = 15$

- (a) Prove that there is infinite number of primes of the form $4k + 3$.

Or

Find the greatest common divisor g of the numbers 1819 and 3587 and find the integers x and y to satisfy $1819x + 3587y = g$.

- (b) Find the solution of the system of congruences

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 2 \pmod{7}$$

- (c) Prove that the equation $x^2 + y^2 = z^2$ has a primitive solution if and only if there exists $s, t \in N$, $s > t$, $(s, t) = 1$ and one even and the other odd such that

$$x = s^2 - t^2$$

$$y = 2st$$

$$z = s^2 + t^2$$

Or

Examine whether the following Diophantine equation has solution. If it is solvable, find the solution :

$$172x + 20y = 1000$$

4. Answer either (a) or (b) :

10

(a) For each positive integer $n \geq 1$, prove that

$$\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d} = n \prod_p \left(1 - \frac{1}{p}\right)$$

The symbols have their usual meanings.

(b) If $n = p_1^{s_1} p_2^{s_2} \dots p_r^{s_r}$, p_i distinct primes and integers $s_i \geq 1$, then for each $r \geq 1$, prove that

$$\tau(n) = (s_1 + 1)(s_2 + 1) \dots (s_r + 1)$$

5. Answer either (a) or (b) :

10

(a) Prove that every Boolean function which does not contain any constant is equivalent to function in disjunctive normal form (DNF).

10

(b) (i) Find the conjunctive normal form (CNF) for the function

$$f = x_1 x_2 x_3 + x_1 x_2' x_3 + x_1' x_2' x_3 + x_1' x_2' x_3'$$

(ii) Design a circuit connecting two switches and one bulb in such a way that either switch may be used to control the light independent of the order.

5+5=10

6. Answer either (a) or (b) : 10

(a) \mathcal{A} is a statement form which contains another statement form \mathcal{A}_1 . Further suppose \mathcal{B} is obtained from \mathcal{A} by substituting \mathcal{B}_1 for one or more occurrences \mathcal{A}_1 , then show that $(\mathcal{A}_1 \leftrightarrow \mathcal{B}_1) \rightarrow (\mathcal{A} \leftrightarrow \mathcal{B})$ is a tautology. Hence if \mathcal{A}_1 and \mathcal{B}_1 are logically equivalent, then show that \mathcal{A} and \mathcal{B} are logically equivalent. 10

(b) (i) Find the statement form in the connectives \sim , \wedge and \vee that generates the function f , where

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
T	T	T	T
T	F	T	T
T	T	F	T
T	F	F	F
F	T	T	F
F	F	T	F
F	T	F	F
F	F	F	T

5

(ii) Show that the statement form $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology. 5
