

2014

MATHEMATICS

(Major)

Paper : 6.2

(Numerical Analysis)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×7=7
- (a) What is the normalized floating point representation of real numbers?
 - (b) Define the term relative error.
 - (c) If we take $\pi = 3.14$ instead of 3.14159 , find the absolute error.
 - (d) Establish the relation, $E \approx 1 + \Delta$.
 - (e) Write down the value of $\Delta^{n+1}x^n$.
 - (f) When are Newton's interpolation formulae used?
 - (g) What is the basic principle of numerical differentiation?

2. Answer the following questions : 2×4=8

(a) Using normalized floating point representation, add $.4546E5$ and $.5433E7$.

(b) Show that

$$\Delta\{\log f(x)\} = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$

(c) Establish the result

$$y' = \frac{dy}{dx} = \frac{1}{h} \left[\Delta y - \frac{1}{2} \Delta^2 y + \frac{1}{3} \Delta^3 y - \frac{1}{4} \Delta^4 y + \dots \right]$$

where the symbols have their usual meanings.

(d) Write the conditions under which
(i) trapezoidal rule and (ii) Simpson's $\frac{1}{3}$ rd rule are valid.

3. Answer the following questions : 5×3=15

(a) Explain the terms truncation error and round-off error with suitable examples.

(b) Using the method of separation of symbols, prove that

$$u_1 x + u_2 x^2 + u_3 x^3 + \dots = \frac{x}{1-x} u_1 +$$

$$\frac{x^2}{(1-x)^2} \Delta u_1 + \frac{x^3}{(1-x)^3} \Delta^2 u_1 + \dots$$

(3)

Or

Obtain the estimate of the missing figures in the following table :

x	:	1	2	3	4	5	6	7	8
$f(x)$:	2	4	8	—	32	—	128	256

(c) Find the first derivative of the function tabulated below at the point $x = 3.0$:

x	:	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$:	-14.000	-10.032	-5.296	0.256	6.672	14.000

Or

Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) Simpson's $\frac{1}{3}$ rd rule and (ii) Simpson's $\frac{3}{8}$ th rule.

4. Answer either (a) or (b) :

(a) (i) Derive the Newton's forward interpolation formula and mention when the formula gives best approximation.

$$4+1=5$$

- (ii) In an examination, the number of candidates who obtained marks between certain limits were as follows :

Marks obtained	Number of candidates
0-19	41
20-39	62
40-59	65
60-79	50
80-99	17

Estimate the number of candidates who obtained less than 70 marks. 5

- (b) (i) State and derive Stirling's central difference formula. Hence or otherwise establish Bessel's formula. 3+3=6

- (ii) Apply Bessel's formula to obtain y_{25} , given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$. 4

5. Answer either (a) or (b) :

- (a) Explain briefly the idea of numerical integration. Establish the general quadrature formula and deduce trapezoidal rule from it. 2+5+3=10

- (b) (i) The velocity v (km / min) of a vehicle which starts from rest, is given at fixed intervals of time t (min) as follows :

t	:	2	4	6	8	10	12	14	16	18	20
v	:	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes, using Simpson's $\frac{1}{3}$ rd rule. 5

- (ii) Compute the integral $\int_5^{12} \frac{1}{x} dx$ by applying Gauss's quadrature formula. 5

6. Answer either (a) or (b) :

- (a) (i) State the Newton-Raphson formula and give a geometrical interpretation of it. 5
- (ii) Find a root of the equation $x^3 - 4x - 9 = 0$, using the bisection method correct to four decimal places. 5
- (b) (i) State the condition of convergence of Newton-Raphson method. 1

(ii) Derive $x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right)$ for determining the square root of $a > 0$, using Newton-Raphson formula. 2

(iii) Show that the equation $x^2 + \log x = 0$ has exactly one root and the root lies in the interval $[\frac{1}{3}, 1]$. 2

(iv) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places. 5
