Total No. of printed pages = 6

3 (Sem 6) MTH M5

2015

MATHEMATICS

(Major)

Theory Paper : M-6.5

(Graph and Combinatorics)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

(a) How many functions are there from a set with 3 elements to a set with 5 elements ?

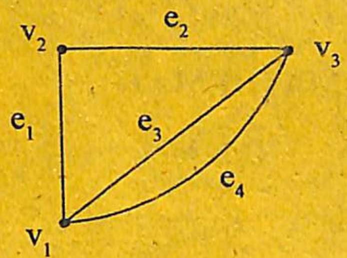
(b) How many ways are there to draw a club or a spade from a pack of cards ?

(c) Draw a simple graph having four vertices each of degree two.

(d) Define the union of two graphs.

(e) Determine if the walk

(v_2, e_2, v_3, e_3, v_1, e_4, v_3) is a path



(f) What is meant by the length of a walk ?

(g) Is the following statement true ?

"In any graph, the number of odd vertices is even."

2. Answer the following questions : 2×4=8

(a) State the rule of sum in the theory of counting.

(b) Find the number of subsets of the set

$\{1, 2, 3, 4, \ldots\ldots, n\}$.

(d) Find all integral solution of the following linear Diophantine equation $8x - 10y = 42$.

3. Answer the following questions : $5 \times 3 = 15$

(a) For any integer $n \geq 2$, if p divides $a_1, a_2 \ldots a_n$, then prove that p divides one of the integers $a_1, a_2, \ldots, a_n$, where p is a prime number. Applying this result, show that 12 is not a prime number.

Or

If p_n is the nth prime, then prove that

$\frac{1}{p_1} + \frac{1}{p_2} + \ldots\ldots + \frac{1}{p_n}$ is not an integer.

(b) Solve the linear congruence

$$6x \equiv 15 \pmod{21}$$

Or

Determine the integer in the unit place of $17^{17^{17}}$.

(c) If $p^c | n$, $p^{c+1} \nmid n$ where p is a prime of the form $4k + 3$ and c is odd, then prove that n has no representation as the sum of two squares.

4. (a) Answer either (i) or (ii) : 10

(i) (1) If the integer $n > 1$ has the prime factorization

$n = p_1^{k_1} p_2^{k_2} \ldots\ldots p_r^{k_r}$ then show that

$$\tau(n) = \prod_{i=1}^{r} (k_i + 1).$$

Hence show that τ is a multiplicative function. 5

(2) Define Mobius μ function. Show that μ is a multiplicative function. If n is a positive integer such that $n \geq 3$, show that

$$\sum_{k=1}^{n} \mu(k!) = 1.$$ 3+2=5

(ii) Define Euler's phi-function. Find $\phi(20)$. If p is a prime and n is a positive integer, then prove that

$$\phi(p^n) = p^n\left(1 - \frac{1}{p}\right).$$ 1+2+7=10

(b) Answer either (i) or (ii) : 10

(i) (1) Let p be "6 is a real number", q be "2+4 = 9" and r be "sum of two even integers is even". Then find the truth value of the following statement forms : 5

(i) $p \rightarrow (q \wedge r)$

(ii) $(p \wedge q) \vee (p \wedge r)$

(iii) $(p \rightarrow (\sim q \vee r)) \wedge \sim (q \vee (p \leftrightarrow r))$

(2) State the principle of substitution. Using the principle, show that the following statements formula is a tautology :

$$(p_1 \wedge \sim p_2) \rightarrow ((\sim p_3 \wedge p_4) \rightarrow ((p_1 \wedge \sim p_2) \wedge (\sim p_3 \wedge p_4)))$$ 5

(ii) (1) Using truth table, verify the following :

$$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$$ 5

(2) Write the truth tables for the connectives 'NOR' and 'NAND'. Show that each of the connectives alone forms an adequate system. 5

(c) Answer either (i) or (ii) : 10

(i) (1) Express the following Boolean expression in disjunctive normal form (DNF) and conjunctive normal form (CNF) :

$$(x + y + z)\ (xy + x'z)'$$ 5

(2) Find a switching circuit which realizes the switching function f given by the following switching table : 5

x	y	z	f (x, y, z)
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

Or

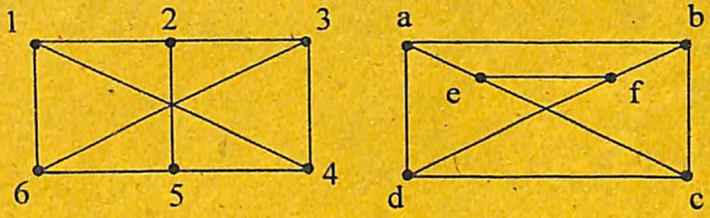
Prove that a connected graph G remains connected after removing an edge e from G if and only if e is in some cycle in G. 5

(b) Define

(i) a bridge in a graph,

(ii) a separable graph. 1+1=2

(c) Define graph isomorphism. Examine if the following two graphs display an isomorphism between them. 1+2=3



6. (a) (i) Define an Eulerian graph and a Hamiltonian graph.

(ii) Give an example of a graph which is Hamiltonian, but not Eulerian.

(iii) Give an example of a graph which is Eulerian but not Hamiltonian.

1+1+1+1=4

(b) If a connected graph G is Eulerian, prove that every vertex of G has even degree. 6

Or

Prove that there is always a Hamiltonian path in a directed complete graph. 6