2014

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatorics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) If A and B are two disjoint events where A occurs in m ways and B occurs in n ways, then in how many ways does the event A or B occur?

(b) How many ways are there to pick an ace or a queen from a deck of cards?

(c) A cubic graph is a —— graph. planar regular

(Fill in the blank)

(d) What is the degree of each point of a complete graph K_5? $5-1=4$

(e) Define cutpoint of a graph G.

(f) What is the connectivity of a connected graph with a cutpoint?

(g) Give an example of a graph which is Hamiltonian but not Eulerian.

2. Answer the following questions : 2×4=8

(a) State the rule of product.

(b) How many ways are there to deal a red ace and then another red card from a deck?

(c) Define self-complementary graph.

(d) Let $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ be two graphs having disjoint point sets and line sets. Find the number of points and number of lines of $G_1 + G_2$.

3. Answer any *three* parts : 5×3=15

(a) Show that the number of r-sequences from n objects is n^r. 5

(b) Show that a graph G is a tree if and only if every pair of points is connected by a unique path. 5

(Continued)

(c) Show that among all graphs with p points and q lines, the maximum connectivity is 0, when $q < p-1$ and is $[2q/p]$, when $q \geq p-1$, where $[r]$ denotes the greatest integer not exceeding the real number r. 5

(d) Give an example of a graph which is—

(i) both Eulerian and Hamiltonian;

(ii) Eulerian but non-Hamiltonian.

2½+2½=5

(e) (i) Does there exist a connected acyclic graph with 10 points and 8 lines? Justify.

(ii) Does there exist a tree with six points having degrees 1, 3, 4, 4, 6? Justify. 2½+2½=5

4. (a) How many non-negative integer solutions are there to—

(i) $X_1 + X_2 + X_3 + X_4 \leq 99$;

(ii) $2X_1 + X_2 + X_3 = 4$ with $X_i \geq 0$? 5+5

Or

(b) (i) What is the probability that a role of three distinct dice produces a sum of ten? 5

(ii) Write equivalent integer-solution of an equation problem for the following : 5

(1) The number of ways to distribute r identical balls into n distinct cells with at least k balls in the first cell

(2) The number of ways to distribute r identical balls into n distinct cells so that no cell contains more than two balls

5. (a) Define intersection graph with suitable examples. Let G be a connected graph with $p > 3$ points. Show that $W(G) = q$ if and only if G has no triangles (where the symbols have their usual meanings). 2+8

Or

(b) Define (i) a non-separable graph, (ii) a block of a graph and (iii) a bridge in a graph. Show that if G is a block, then—

(i) every two points of G lie on a common cycle;

(ii) every point and line of G lie on a common cycle. 3+7

(Continued)

6. *(a)* Show that the following statements are equivalent for a connected graph G : 10

(i) G is Eulerian

(ii) Every point of G has even degree

(iii) The set of lines of G can be partitioned into cycles

Or

(b) Let G have $p \geq 3$ points. If for every n, $1 \leq n < (p-1)/2$, the number of points of degree not exceeding n is less than n and if for odd p, the number of points of degree at most $(p-1)/2$ does not exceed $(p-1)/2$, then show that G is Hamiltonian.

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