Total No. of printed pages = 7

3(Sem 6) MTH M4

2015

MATHEMATICS

(Major)

Theory Paper : M-6.4

(Discrete Mathematics)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed. $1 \times 7 = 7$

(a) If n is a positive integer such that n^3+1 is a prime, then find the value of n.

(b) For all integers $n \geq 0$, $7^n - 1$ is divisible by 6. (State whether true or false).

[Turn over

(c) State Euclid's theorem on prime numbers.

(d) Find all integers $k \geq 2$ such that $7 \equiv k \pmod{k^2}$.

(e) If n is a positive integer such that gcd $(n, 9) = 1$, then $n^{18} - 1$ is not divisible by 9. (State whether true or false).

(f) State Fermat's Little Theorem.

(g) State the condition for which the linear Diophantine equation $ax + by = c$ has an integral solution.

2. Answer the following questions : $2\times4=8$

(a) If a and b are positive integers such that gcd $(a, b) = 1$, then show that

gcd $(a + b, a - b) = 1$ or 2.

(b) Find the remainder when $\lfloor 17$ is divided by 19.

(c) Show that if $x^2 + y^2 = z^2$, then one of x, y is $\pm 1 \pmod 4$ and the other is $0 \pmod 4$.

(c) Define a complete bipartite graph. Draw a complete bipartite graph on 2 and 4 vertices.

(d) Does there exist a simple graph with five vertices having degrees 2, 2, 4, 4, 4 ? Justify.

3. Answer any *three* parts of the following : $3\times5=15$

(a) Give combinational proofs of the following identities : $2+3=5$

(i) $C(n, r) = C(n, n-r)$

(ii) $C(n+1, r) = C(n, r) + C(n, r-1)$

(b) (i) How many selections can be made from 3 white balls, 4 green balls, 1 red ball, 1 black ball ; if at least one must be chosen ?

(ii) In how many ways can a person invite one or more of his 5 friends to a party ? $3+2=5$

(c) (i) Draw the graphs K_4 and $K_{2,3}$.

(ii) How many vertices are there in a graph with 15 edges if each vertex is of degree 3 ? $2+3=5$

(d) Define a path.

If a graph G contains exactly two vertices of odd degree, show that there exists a path between these two vertices. 1+4=5

(e) Define a tree.

If in a graph G, there is a unique path between every pair of vertices, show that G is a tree. 1+4=5

4. (a) How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 16$, where each $x_i \geq 2$? 3

(b) How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$, where $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$? 4

(c) What is the probability that exactly one cell is empty if ten identical balls are distributed randomly into five distinct cells ? 3

5. (a) Prove that a connected graph G with n vertices is a tree if and only if G contains (n − 1) edges. 5

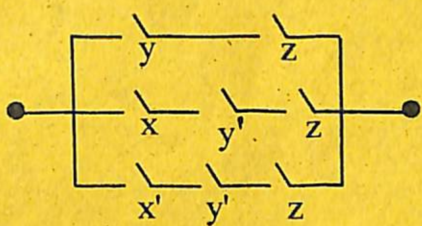
(ii) (1) Simplify the Boolean expression :

$$(x + y)\,(x + z)\,(x'\,y')'$$

Express the following Boolean expression in conjunctive normal form (CNF) in the variables present in the expression :

$x' + yz$. 2½+2½=5

(2) Consider the following switching circuit :



Find a Boolean expression which represents the circuit. Also, draw a simpler equivalent circuit for the above circuit. 3+2=5