2014

MATHEMATICS

(Major)

Paper : 4.2

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) What is the moment of a force about a point lying on its line of action?

(b) Under what situation two couples are equivalent to each other?

(c) Write down the equation to the resultant of a system of forces in one plane.

(d) Define the centre of gravity of a body.

(e) Define Poinsot's central axis.

(f) What are the elements of a system of forces acting on a rigid body?

(g) What do you mean by stable equilibrium of a body?

(h) Write down the expression of acceleration of a moving point in polar coordinate system.

(i) What is the geometrical representation of the simple harmonic motion?

(j) What are the characteristics of a central force?

2. Answer the following questions : 2×5=10

(a) If two like parallel forces P and Q acting on a rigid body at A and B be interchanged in position, show that the point of application of the resultant will be displaced along AB through a distance d, where

$$d = \frac{P-Q}{P+Q} \cdot AB, \quad (P > Q)$$

(b) Show that the centre of gravity of a body is unique.

(c) Find the centre of gravity of an arc of a plane curve, $y = f(x)$.

(d) A particle describes an ellipse under a force $\frac{\mu}{r^2}$ and has a velocity v at a distance r from the centre of force. Show that its periodic time is

$$\frac{2\pi}{\sqrt{\mu}}\left(\frac{2}{r}-\frac{v^2}{\mu}\right)^{-3/2}$$

(e) Show that the conservation of energy for a particle of mass m in a central force field can be expressed as

$$\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-\int f(r)\,dr = E$$

where E is a constant.

3. Answer the following : 4×5=20

(a) State the forces which may be omitted in forming the equation of virtual work.

(b) Equal forces act along the coordinate axes and along the straight line

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

Find the equations of the central axis of the system.

(c) A hemisphere rests in equilibrium on a sphere of equal radius. Show that the equilibrium is unstable when the curved surface rests on the sphere and stable when the flat surface of the hemisphere rests on the sphere.

(d) A particle is moving in a plane curve. Find the components of its acceleration along the tangent and the normal to the curve at any instant.

(e) The position of a particle moving in a straight line is given by

$$x = a\cos nt + b\sin nt$$

Prove that it executes simple harmonic motion and amplitude is $\sqrt{a^2 + b^2}$.

Or

Two bodies M and M' are attached to the lower end of an elastic string whose upper end is fixed and are hung at rest, M' falls off. Show that the distance of M from the upper end of the string at time t is

$$(a+b) + c\cos\left(\sqrt{\frac{g}{b}}t\right)$$

where a is unstretched length of the string, b and c are the distances by which it would be executed when supporting M and M' respectively.

4. Answer the following : 5×4=20

(a) ABC is an equilateral triangle of side a; D, E, F divide sides BC, CA and AB in the ratio 2 : 1. Three forces each equal to P act at D, E, F perpendicular to the sides and directed away from the triangle. Prove that they are equivalent to a couple of moment $\frac{1}{2}Pa$.

(b) Find the position of CG of the area enclosed by the curves $y^2 = 4ax$ and $y = mx$.

Or

Find the position of CG of a homogeneous solid hemisphere.

(c) A solid hemisphere rests on a plane inclined to the horizon at an angle α

$$\left(< \sin^{-1}\frac{3}{8}\right)$$

and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable.

(d) State the Kepler's laws of planetary motion and briefly mention their dynamical significances.

5. *(a)* Define limiting friction and coefficient of friction. 2+2=4

(b) A ladder whose centre of gravity divides it into two portions of lengths a and b with one end on a rough horizontal floor and the other against a rough vertical wall. If the coefficients of friction at the floor and at the wall be μ and μ' respectively, find its inclination to the floor when the equilibrium is limiting. 6

6. *(a)* A particle is projected along the inner surface of a rough sphere and is acted on by no force. Show that it will return to the point of projection after time

$$\frac{a}{\mu V}(e^{2\pi\mu} - 1)$$

where a is the radius of the sphere, V is the velocity of projection and μ is the coefficient of friction. 5

(b) Prove that a particle, projected upwards with a velocity U in a medium whose resistance varies as the square of the velocity, will return to the point of projection with velocity

$$v = \frac{UV}{\sqrt{U^2 + V^2}}$$

where V is the terminal velocity. 5

Or

Establish the formula

$$F = m\frac{dv}{dt} + \lambda u$$

for the motion of a particle of varying mass $m(t)$ with velocity v under a force F, matter being emitted at a constant rate λ with velocity u relative to the particle.

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