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MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following : 1×10=10

(a) What does the singular solution of a differential equation represent?

(b) Give one example of a linear differential equation.

(c) Write the complementary function of
 $(D^2 + 4)y = x^2 \sin 2x$

(d) Write the form of a total differential equation.

(e) What does the complete integral of a first-order partial differential equation represent?

(f) Give an example of a first-order and second-degree differential equation.

- (g) When a total differential equation is said to be exact?
- (h) Define linear partial differential equation.
- (i) What is an ordinary differential equation?
- (j) Write down the order and degree of

$$x^2 \left(\frac{d^2 y}{dx^2} \right)^3 + y \left(\frac{dy}{dx} \right)^4 + y^4 = 0$$

2. Answer any *five* of the following questions :

2×5=10

- (a) Prove that $y = \sin x$ is a part of complementary function of
 $(\sin x - x \cos x)y'' - x \sin x y' + y \sin x = 0$
- (b) Give a geometrical interpretation of the equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, where P, Q, R are the functions of x, y, z .
- (c) Find the differential equation of the family of curves $y = me^{2x} + ne^{-2x}$ for different values of m and n .
- (d) Solve $\frac{dy}{dx} = \sec(x+y)$.
- (e) Find a partial differential equation by eliminating a and b from $az + b = a^2 x + y$.
- (f) Solve $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$.

3. Answer any five of the following questions :

4×5=20

(a) Reduce $y = 2px + y^2 p^3$ to Clairaut's form using the transformation $y^2 = v$ and hence solve it.

(b) Solve $(D^2 - 2D + 1)y = \cos 3x$.

(c) Solve $(ax + hy + g)dx + (hx + by + f)dy = 0$.

(d) Solve $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2 + y^2}$.

(e) Solve by the method of variation of parameters $\frac{d^2 y}{dx^2} + a^2 y = \sec ax$.

(f) Solve :

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2 \cos t - 7 \sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4 \cos t - 3 \sin t$$

4. Answer any five of the following questions :

6×5=30

(a) Solve $\frac{d^4 y}{dx^4} + m^4 y = 0$.

(b) Verify the condition of integrability for $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$ and solve it.

(c) Solve $z(z^2 + xy)(px - qy) = x^4$.

- (d) Find a complete integral of

$$z^2 (p^2 z^2 + q^2) = 0$$

by Charpit's method.

- (e) Form a partial differential equation by eliminating the arbitrary function
- ϕ
- from

$$\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$$

- (f) Find
- $f(y)$
- s.t.
- $\{(yz + z)/x\} dx - zdy + f(y)dz = 0$
- is integrable. Also find the corresponding integral.

5. Answer either (a) and (b) or (c) and (d) : 5+5=10

- (a) Solve
- $x^2 y'' + xy' - y = 0$
- , given that
- $x + \frac{1}{x}$
- is one integral.

- (b) If
- u
- and
- v
- are two independent particular integrals of the equation

$$\frac{d^2 y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = 0$$

prove that $u \frac{dv}{dx} - v \frac{du}{dx} = c \cdot e^{-\int P dx}$.

- (c) Solve
- $y'' \cos x + y' \sin x - 2y \cos^3 x = 2 \cos^5 x$
- by changing the independent variable.

- (d) Find the equation of the integral surface of the differential equation

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

which passes through the lines $x=1$, $y=0$.
