2017

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×10=10

   (a) Determine the order and degree of the differential equation

   $$K\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

   (b) Define Bernoulli's differential equation.

   (c) If the differential equation $Mdx+Ndy=0$ is homogeneous and $Mx+Ny\neq 0$, write the integrating factor.

   (d) What do you mean by self-orthogonal family of curves?

(e) What is the complementary function of the following differential equation?

$$(D^3 + 3D^2 + 3D + 1)y = e^{-x}$$

(f) Write down the general solution of the differential equation

$$y = px + p - p^2$$

(g) Write down the condition of exactness of a total differential equation

$$Pdx + Qdy + Rdz = 0$$

(h) Find the particular integral of the differential equation

$$(D^2 + a^2)y = \sin ax$$

(i) Write the standard form of the linear partial differential equation of order one.

(j) Find an integral belonging to complementary function of the differential equation

$$y_2 - \cot x y_1 - (1 - \cot x)y = e^x \sin x$$

2. Answer the following questions : 2×5=10

(a) Form the differential equation of which $xy = ae^x + be^{-x}$ (a, b parameters) is a solution.

(b) Solve :

$$(x+y)^2 \frac{dy}{dx} = a^2$$

(c) Solve :

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2 y^2 z^2}$$

(d) If $\frac{dy}{dx} + 2y \tan x = \sin x$ and if $y = 0$ when $x = \frac{\pi}{2}$ express y in terms of x.

(e) Construct the partial differential equation by eliminating a and b from

$$z = (x^2 + a)(y^2 + b)$$

3. Answer any *four* questions : 5×4=20

(a) Prove that a necessary and sufficient condition that the differential equation $Mdx + Ndy = 0$ be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(b) Find the orthogonal trajectories of the family of curves $x^2 + y^2 = 2ax$, a being parameter.

(c) Solve :

$$\frac{dy}{dx} = \frac{3y - 7x + 7}{3x - 7y - 3}$$

(d) Apply variation of parameter to solve the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$$

(e) Solve :

$$x^3 \frac{d^3y}{dx^3} + 6x^2 \frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + 2y = x^2 + 3x - 4$$

(f) Obtain the general and singular solution of the differential equation

$$y = px + \sqrt{b^2 + a^2 p^2}$$

4. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Solve :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$$

(b) Solve :

$$t\,dx = (t - 2x)\,dt$$
$$t\,dy = (tx + ty + 2x - t)\,dt$$

(c) Solve :

$$\frac{dy}{dx} = x^3 y^3 - xy$$

(d) Reduce the equation $y^2(y-px)=x^4p^2$, where $p=\dfrac{dy}{dx}$ to Clairaut's form by the substitution $x=\dfrac{1}{X}$, $y=\dfrac{1}{Y}$ and hence solve the equation.

5. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Find the necessary condition for integrability of the total differential equation $Pdx+Qdy+Rdz=0$.

(b) Reduce the differential equation

$$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=R(x)$$

to the form $\dfrac{d^2v}{dx^2}+Q_1v=R_1$, where Q_1 and R_1 are functions of x to solve the differential equation.

(c) Solve :

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin x^2$$

by changing the independent variable x to z.

(d) Solve :

$$(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x(1-x^2)^{3/2}$$

6. Answer *either* *(a)* and *(b)* or *(c)* and *(d)* : 5+5=1

(a) Solve :

$$\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$$

(b) Solve :

$$3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0$$

(c) Show that the differential equation

$$xdx+ydy=\frac{a^2(x\,dy-y\,dx)}{x^2+y^2}$$

is exact and hence solve it.

(d) Find $f(z)$ such that

$$\left(\frac{y^2+z^2-x^2}{2x}\right)dx-ydy+f(z)dz=0$$

is integrable and hence solve it.

7. Answer *either* *(a)* and *(b)* or *(c)* and *(d)* : 5+5=10

(a) Solve by Lagrange's method

$$z(x+y)p+z(x-y)q=x^2+y^2$$

(b) Solve by Charpit's method

$$pxy+pq+qy-yz=0$$

(c) Find the complete integral of $9(p^2z+q^2)=4$. Also find the singular solution if it exists.

(d) Derive the partial differential equation by the elimination of arbitrary function from the equation $\phi(u, v)=0$, where u and v are functions of x, y and z.

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