2017

MATHEMATICS

(Major)

Paper : 2·1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Transform to axes inclined at 45° to the original axes the equation $x^2 - y^2 = a^2$.

(b) Write down the condition for pair of lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

(c) Write down the parametric equations of the parabola.

(d) Write down the direction cosines of x-axis.

(e) About which axis the parabola $y^2 = 4ax$ is symmetric?

(f) Find the eccentricity of the ellipse

$$x^2 + 3y^2 = a^2$$

(g) Write the equation of the diameter of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

parallel to the line $y = mx + c$.

(h) Find the centre and radius of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y + 2z + 3 = 0$$

(i) Define conjugate planes.

(j) Define enveloping cylinder.

2. Answer the following : 2×5=1

(a) If the axes be turned through an angle $\tan^{-1} 2$, what does the equation $4xy - 3x^2 = a^2$ become?

(b) Find the value of k so that $kxy - 8x + 9y = 12$ may represent pair of straight lines.

(c) Find the equation of the sphere through circles

$$x^2+y^2+z^2=25, \quad x+2y-z+2=0$$

and the point (1, 1, 1).

(d) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola $y^2=4ax$, prove that $t_1t_2=-1$.

(e) The axis of a right circular cylinder is

$$\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z-3}{2}$$

and its radius is 5. Find the equations.

3. Answer any *two* parts : 5×2=10

(a) By transforming to parallel axes through a properly chosen point (h, k), prove that the equation

$$12x^2-10xy+2y^2+11x-5y+2=0$$

can be reduced to one containing only terms of the second degree.

(b) Reduce the equation

$$7x^2-2xy+7y^2-16x+16y-8=0$$

to the standard form.

(c) Show that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight lines, if

$$\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$$

(d) Prove that the sum of the squares of the reciprocals of two perpendicular diameters of an ellipse is constant.

4. Answer any *two* parts : 5×2=10

(a) Prove that the straight line $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$, if

$$c = ma + \frac{a}{m}$$

(b) Find the length of the semi-axes of the conic $ax^2 + 2hxy + ay^2 = d$.

(c) Prove that the line $lx + my = n$ is a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

if $\dfrac{a^2}{l^2} + \dfrac{b^2}{m^2} = \dfrac{(a^2 - b^2)^2}{n^2}$.

(d) Find the equation of the tangent to the hyperbola $4x^2 - 9y^2 = 1$ which is parallel to the line $4y = 5x + 7$.

5. Answer any *four* parts : 5×4=20

(a) Find the condition that the homogeneous equation of second degree

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

may represent a pair of planes.

(b) Obtain the shortest distance between the lines

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

and

$$\frac{x-\alpha'}{l'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$$

(c) Prove that the equation of the plane containing the line

$$\frac{y}{b} + \frac{z}{c} = 1, \quad x = 0$$

and parallel to the line $\frac{x}{a} - \frac{z}{c} = 1$, $y = 0$ is

$$\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$$

If $2d$ is the shortest distance between the given lines, prove that

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{d^2}$$

(d) Find the condition when the plane $lx + my + nz = p$ becomes a tangent plane to the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

(e) Find the equation of the cone whose vertex is at the origin.

(f) Prove that the equation of the polar of the origin with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

is $gx + fy + c = 0$.

6. Answer any *four* parts : 5×4=20

(a) Find the equation of the cylinder whose axis and guiding curve are given.

(b) Find the condition when the plane $lx + my + nz = p$ becomes a tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$.

(c) Find the equation of the enveloping cone of a conicoid whose vertex is given.

(d) If the section of the enveloping cone of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

whose vertex is P, by the plane $z = 0$ is a rectangular hyperbola, prove that the locus of P is

$$\frac{x^2 + y^2}{a^2 + b^2} + \frac{z^2}{c^2} = 1$$

(e) Prove that the locus of the poles of tangent plane of the conicoid $ax^2 + by^2 + cz^2 = 1$ with respect to the conicoid $\alpha x^2 + \beta y^2 + \gamma z^2 = 1$ is the conicoid

$$\frac{\alpha^2 x^2}{a} + \frac{\beta^2 y^2}{b} + \frac{\gamma^2 z^2}{c} = 1$$

(f) Find the equation of the right circular cylinder whose guiding curve is

$$x^2 + y^2 + z^2 = 9$$
$$x - y + z = 3$$

★★★